Open Access

SCHOLARENA

Journal of Nanoscience and Nanotechnology Applications

RESEARCH ARTICLE

ISSN: 2577-7920

Prevention Strategies for Mathematical Model MERS-Corona Virus with Stability Analysis and Optimal Control

Tahir M[1*], Ali Shah IS[1], Zaman G[2] and Khan T[2]

[1]*Department of Mathematics, Islamia College Peshawar, K P K Pakistan*
[2]*Department of Mathematics, University of Malakand, K P K Pakistan*

[*]**Corresponding author:** Tahir M, Department Of Mathematics, Islamia College Peshawar, 25000 K P K Pakistan, Tel: 923459063508, E-mail: tahirshah08@yahoo.com

Citation: Tahir M, Ali Shah IS, Zaman G, Khan T (2018) Prevention Strategies for Mathematical Model MERS-Corona Virus with Stability Analysis and Optimal Control. J Nanosci Nanotechnol Appl 3: 101

Article history: Received: 25 October 2018, **Accepted:** 11 January 2019, **Published:** 13 January 2019

Abstract

In this subsection of the article, we considered the transmission of MERS-CoV in human population though an agent known as camel, which lead an epidemiological mathematical model. For this, first we find the threshold number "R_0" of the model, and biological region of study. Then all endemic equilibrium points are derived. After that stability analysis are retrieved in the presence of R_0, while for "global stability analysis" we introduced Lyapunov function. Then to control the infection further we used the control strategy by applying two control variables, u_1, and u_2, that is, using mask to cover full body parts, and medication or antivirus to maximized the number of susceptible individuals as well as recovered individuals, and trying to reduced the number of infected individuals. Finally numerical simulation is presented with and without control.

Keywords: Mers-corona virus; Mathematical Model; Basic Reproduction Number; Endemic equilibrium points; Local Stability Analysis; Global Stability Analysis; Optimal control; Numerical Simulation

Introduction and Related literature

Over then 2,000 cases of the MERS-CoV have been recently reported by 2017, while rate of fatality is > 30 [1]. Eighteen more cases were reported in early May 2018 [2]. After a period of few cases, cases began increasing in the middle of the summer 2018 [3,4]. The earliest cases of MERS were of clade a clusters (EMC/2012 and Jordan-N3/2012), and new cases are genetically distinct (clade B) [5,6]. Early reports compared the virus to severe acute respiratory syndrome (SARS), and it has been referred to as Saudi Arabia's SARS-like virus [7-9]. MERS-CoV is considered zoonotic virus, which means it transferable through animals to humans. Products of camel is the main source of infection in human. The findings suggest that bats may transfer the virus to camels, and camels transmit it to humans. The study up to February 24, 2015, shown there are 1046 cases reported globally the Middle East Respiratory Syndrome Corona virus (MERS-COV) in which 298 of fatal cases is detected. Hence MERS-COV is still gaining power and grounds in the world. In the United Arab Emirates, there is 67 confirmed cases with 9 death cases have been reported. This situation requires the mobilization of all scientific communities to investigate all the possibilities to control the outbreak and try to respond to the many un answered question associated with the explosion of MERS cases in recent weeks [10,11]. The mathematical epidemiology has shown its efficiency in shedding light on several issues related to the spread of diseases. Severe Acute Respiratory Syndrome (SARS) is the best example, where the Canadian government created the MITACS (Mathematics of Information Technology and Complex System) network to help in the management of SARS epidemic in 2002-2003 [12-14]. Also Tahir, et al. presented the mathematical model for Ebola virus [15]. There is also a report of a Saudi Arabian man who became ill seven days after applying topical medicine to the noses of several sick camels and later he and one of the camels were found to have identical strains of MERS-CoV [16,17]. In January 2018 a larger outbreak of MERS among camels in Kenya is reported [18,19]. As of 5 February 2018 more than 500 camels are said to have died of the said disease [17,20]. MERSE Corona was first occurred in Saudi Arabia in 2012. Corona-viruses having big family viruses which cause infection from a common cold to severe Acute Respiratory Syndrome (SARS). General MERS CoV have fever, cough also shortness of the breath. While Pneumonia considered common symptom, but not always. While diarrhoea is present in gastrointestinal problem. Perform hand hygiene before and after contact with the person and his or her surroundings and after PPE removal [21].

The cells MERS-CoV infects in the lungs only account for 20 percent of respiratory epithelial cells, so a large number of virions are likely needed to be inhaled to cause infection [22,23]. Unless there a close connection among peoples and camels. The

most outbreaks in health occurred in various countries, while most outbreaks observed in Saudi Arabia second United Arab Emirates third in Republic of Korea. Severe illness can cause respiratory failure in human. When MERS patients are admitted to hospitals their clinical symptoms mostly include fever, cough, expectoration and shortness of breath [24,25]. In this article, we processed as follows, first section is about introduction and related literature, then we given the mathematical modeling and all its infectious classes. Then the basic key value threshold number R_0, as well as, the endemic equilibrium points are obtained. In the light of basic reproductive number all equilibria has been calculated and shown stable while in global stability analysis we introduced "Lyapunov" function. Also we used control strategy to maximize the number of susceptible and recovered individuals by introducing two control variables. In the last we gave numerical simulation with and without control or vaccination to represent the model graphically.

Formation of Mathematical Model

Our model is based on SIR human population model, and vector camel population, properties of the disease, and infectious classes. We divided the whole population as.

1. $T_{hp}(t)$ represent human population.
2. $T_{cp}(t)$ represent vector camel population.
3. $S(t)$ represent susceptible human population .
4. The infected human population represent by $I(t)$ which are infected by infected camel through (close contact, caring, treatment, using infected camel milk, meat, saliva) are the causes of infection.
5. $R(t)$ represented recovered individuals population.

Now the vector camels population is described as,

6. Susceptible camels population is represented by $S_c(t)$.
7. Infected camels population is represented by $I_c(t)$ respectively.

Here we drawn some compartments in the model as,

8. During care, close contact the susceptible human individuals (owner, shepherd, cherisher) infected partially and move from susceptible human individual compartment1 to infected human individual compartment 4.
9. The partially infected individual using medication or conservancy injection and move from that compartment to recovered individual compartment 5 or die.
10. Totally infected individual joining hospital and spread infection there or die.

First we represent human population at any time "t" while in second we represent camel population in the same time. Then from all 1 to 10 we get the following mathematical model of Mers-Corona virus which lead the following differentials equations:

Human population:

$$\begin{aligned} S\bullet &= \chi + \beta_1 SI - \mu_h S - \beta_3 SI + \beta_2\beta_3 SI. \\ I\bullet &= \beta_1 SI + \beta_3 SI - (\mu_1 + \mu_2)I - \beta_2 I + \beta_3 IR. \\ R\bullet &= \beta_2 I - \mu R + \beta_2 \chi. \end{aligned}$$

Vector camel population is:

$$\begin{aligned} Sc\bullet &= \mu_c - \beta_4 ScIc - \mu_{sc} Sc + \beta_4\beta_2(\mu_1 + \mu_2)Ic. \\ Ic\bullet &= \beta_4 ScIc - (\mu_3 + \mu_4)Ic + \beta_4 ScIc. \end{aligned}$$

With the following initial conditions,

$S(0) \geq 0, I(0) \geq 0, S_c(0) \geq 0, I_c(0) \geq 0.$

further we used the following assumptions in model (1), which are classified as, x represent new birth rate in susceptible human population, β_1 represent the transmission rate from susceptible human individuals to infected human individuals due to infection of infected camel and move from susceptible human compartment to infected human individual compartment, represent natural death rate in susceptible human individuals compartment, β_2 represent the rate of transmission of partially infected individual to recovered human individuals compartment, μ_1 and μ_2 respectively represent natural death rate and infected death rate in infected human individuals compartment, μR represent natural death rate in recover human individuals compartment, μc represent new birth rate in susceptible camels population compartment, μsc represent natural death rate in susceptible camels population, β_4 represent infection rate in susceptible camels to become infected camel compartment, μ_3 and μ_4 are respectively represent natural and infectious death rate of infected camels population compartment.

In the model the infection spread in susceptible human individual due to the infected camels in any like, blood, wound, saliva, caring, treatment of infected camel especially the individuals like owner of camels, shepherd, cherisher get MERS CoV infection and the rate of that infection is represented by β_3

Threshold Number R_0 of the Model

In mathematical epidemiology study the basic reproduction number R_0 considered a primary and initial parameter. R_0 is one of the basic key of mathematical epidemiology [26]. This (R_0) quantity is been obtained by many methods, while an interesting and simple method is the next generation matrix which very useful in the determination of a biologically meaningful formula for a basic reproduction number in the case of continuous time epidemic model, that is in system of differentials equations [27]. For this we processed in proposed model (1) as, the rate of a secondary infectious ғ of the infected class is represented by (I_h, I_c) also the rate of a disease progression V of non-infected class is denoted by (S, R, S)

$$F = \begin{bmatrix} \beta_1 SI + \beta_3 SI \\ \beta_4 SI \end{bmatrix}.$$

And the non infectious class of the model is,

$$V = \begin{bmatrix} -(\mu_1 + \mu_2)I - \beta_2 I \\ -(\mu_3 + \mu_4)Ic\beta_4 ScIc \end{bmatrix}.$$

The inverse of V is represented by,

$$\overline{V} = \begin{bmatrix} \frac{1}{\mu_1+\mu_2+\beta_2} & 0 \\ 0 & \frac{1}{\mu_3+\mu_4} \end{bmatrix}.$$

Hence the required reproduction number, R_0 of the proposed model is given by,

$$R_0 = Max[\frac{\beta_1 S + \beta_3}{\mu_1 + \mu_2 + \beta_2}, \frac{\beta_4 Sc}{\mu_3 + \mu_4 + Sc}].$$

Here for reproductive number we have two possibilities.
First for susceptible human population we have the following,

$$\frac{dT_{hp}}{dt} = \frac{dS}{dt} + \frac{dI}{dt} + \frac{dR}{dt}.$$

After putting values from model (1) we get,

$$\frac{dT_{hp}}{dt} = \chi - \mu_h S - (\mu_1 + \mu_2)I - \mu R.$$

Which can be written as,

$$\lim_{t\to\infty} T_{np}(t) \leq \frac{\chi}{\mu_h}.$$

Now second for susceptible camel population, we have,

$$\frac{dT_{cp}}{dt} = \frac{dS_c}{dt} + \frac{dI_c}{dt}.$$

After putting values from model (1) we get,

$$\frac{dT_{cp}}{dt} = \mu_c - \mu_{sc} - (\mu_3 + \mu_4)I_c.$$

$$\lim_{t\to\infty} T_{cp}(t) \leq \frac{\mu_c}{\mu_{sc}}.$$

Hence model (1) has two eigen values. The first eigen value is concern with the susceptible human population while the second eigen value related to the susceptible camels

Endemic Equilibrium Points of Model

In this subsection, the potential existence of a disease-free equilibrium points is now discussed. Now we find the endemic equilibrium points. For this we know that the points of disease-free equilibrium results to be locally asymptotically stable when the basic reproduction number, that is, (R_0) < 1, while the endemic equilibrium points is locally asymptotically stable when the reproductive number exceeds unity, that is, > 1. Following are the endemic equilibrium points of the concern mathematical model of MERS CoV.

$$\begin{aligned} S_c^\star &= \frac{\mu_3+\mu_4}{\beta_4}, \\ I_c^\star &= \frac{\beta_4\mu_3+\mu_{sc}(\mu_3+\mu_4)}{\beta_4(\mu_3+\mu_4)}, \\ S^\star &= (\frac{\chi}{\mu_h}-(\frac{\mu_1+\mu_2+\beta_2}{\mu_h})I), \\ R^\star &= (\frac{\beta_2}{\mu})I^\star, \\ I^\star &= \frac{\chi-\mu_h S^\star}{\mu_1+\mu_2+\beta_2}. \end{aligned}$$

Stability Analysis of Model

In this subsection, we present the stability analysis of the model (1). The stability analysis having two categories (1) local stability analysis and (2) global stability analysis which are discussed below.

Local Behaviour of the Model at Disease Free Equilibrium

Now we show the local stability analysis of the system, at disease free equilibrium of model
(1), the points are $J(d_f)(\chi/_{\mu h}, 0, 0, \mu_c/\mu_{sc}, 0)$ to assess the local stability analysis of model (1) at disease free equilibrium, the Jacobian matrix at $J(d_f)$,

$$J(d_f)_0 = \begin{bmatrix} -\mu_h & -\beta_1 S & 0 & 0 & -\beta_3 S \\ 0 & \beta_1 S-(\mu_1+\mu_2)-\beta_2 & 0 & 0 & \beta_3 S \\ 0 & 0 & -\mu(\beta_1 S-(\mu_1+\mu_2)-\beta_2) & 0 & 0 \\ 0 & 0 & 0 & -\mu_{sc} & -\beta_4 S_c \\ 0 & 0 & 0 & 0 & \beta_4 S_c-(\mu_3+\mu_4) \end{bmatrix} \quad (2)$$

So the disease free equilibrium implies $I_h = 0$, $R_h = 0$, $I_c = 0$ then we have,
(2) Thus we have the following well known stability results.

Theorem: If $R_0 < 1$, the proposed model (1) remain locally asymptotically stable on the point disease free equilibrium, $D(F)_0 = (S^0, 0, 0, S_c^o, 0)$ while the model is said unstable,
if $R_0 > 1$.

Proof: From the above equation (2) we write that,

$$\begin{aligned} \lambda_1 &= -\mu_h, && (3) \\ \lambda_2 &= \beta_1 S-(\mu_1+\mu_2)-\beta_2, && (4) \\ \lambda_3 &= -\mu(\beta_1 S_h-(\mu_1+\mu_2)-\beta_2), && (5) \\ \lambda_4 &= -\mu S_c, && (6) \\ \lambda_5 &= \beta_4 S_c-(\mu_3+\mu_4). && (7) \end{aligned}$$

(3), (4), (5), (6), (7)
Its cleared from equations (3), (5) and (6) the eigenvalues are negative. Taking equation (4)

$$\begin{aligned}\lambda_2 &= \beta_1 S - (\mu_1 + \mu_2) - \beta_2,\\ \lambda_2 &= (\mu_1 + \mu_2 + \beta_2)(R^0 - 1),\\ or&\\ \lambda_2 &= -(\mu_1 + \mu_2 + \beta_2)(1 - R^0).\end{aligned}$$

The corresponding value $\lambda_2 < 0$ iff $R_0 < 1$. Now considered equation (7) which after some simplification given.
$\lambda_5 = -(\mu_3 + \mu_4)(1-R_0) < 0$ iff $R_0 < 0$,

Now all the eigne values are negative which complete the proof. Hence we say that local stability analysis at disease free equilibrium of the model (1) are asymptotically stable.

Local Stability Analysis of Model at Endemic Equilibrium

We have the following result to prove the local stability analysis of the model at endemic equilibrium stated below,

Theorem: *If $R_0 > 1$ then the model* (1) *is locally asymptotically stable at endemic equilibria, that is at* $V_E = (S^*, I^*, R^*, S_c^*, I_c^*)$ *and unstable otherwise.*

$$J(I_E) = \begin{bmatrix} -\beta_1 I^\star - \mu_h - \beta_3 I_c^\star & -\beta_1 S^\star & 0 & 0 & -\beta_3 S^\star \\ \beta_1 I^\star + \beta_3 I_c^\star & \beta_1 S^\star - (\mu_1 + \mu_2) - \beta_2 & 0 & 0 & \beta_3 S^\star \\ 0 & \beta_2 & -\mu & 0 & 0 \\ 0 & 0 & 0 & -\beta_4 I_c^\star - \mu_{sc} & -\beta_4 S_c^\star \\ 0 & 0 & 0 & \beta_4 I_c^\star & \beta_4 S_c^\star - (\mu_3 + \mu_4) \end{bmatrix}$$

After simplification we get the following, (8)

$$J(I_E) = \begin{bmatrix} -\beta_1 I^\star - \mu_h - \beta_3 I_c^\star & -\beta_1 S^\star & 0 & 0 & -\beta_3 S^\star \\ 0 & A & 0 & 0 & B \\ 0 & 0 & \beta_1 \mu S^\star & 0 & -\beta_3 S^\star \\ 0 & 0 & 0 & -\beta_4 I_c^\star - \mu_{sc} & -\beta_4 S_c^\star \\ 0 & 0 & 0 & 0 & E \end{bmatrix} \quad (8)$$

Here we made the following assumptions in the above which are describe as below,

$$\begin{aligned} A &= -\beta_1 S^\star(\beta_1 I^\star + \beta_3 I_c^\star) - ((\mu_1 + \mu_2 + \beta_2) - \beta_1 S^\star))(\beta_1 I^\star + \mu_h + \beta_3 I_c^\star),\\ B &= -\beta_3 S^\star(\beta_1 I^\star + \beta_3 I_c^\star) - \beta_3 S^\star(\beta_1 I_h^\star + \mu_h + \beta_3 I_c^\star),\\ E &= -(\beta_4^2 S_c^\star I_c^\star) + [(\mu_3 + \mu_4) - \beta_4 S_c^\star)(\beta_4 I_c^\star + \mu_{sc})].\end{aligned}$$

From equation (8) we get the following Eigne values which are listed below,

$$\begin{aligned} \lambda_1' &= -(\beta_1 I^\star + \mu_h + \beta_3 I_c^\star), && (9)\\ \lambda_2' &= A, && (10)\\ \lambda_3' &= \beta_1 \mu S^\star, && (11)\\ \lambda_4' &= -(\beta_4 I_c^\star + \mu_{sc}), && (12)\\ \lambda_5' &= E. && (13)\end{aligned}$$

(9), (10), (11), (12), (13)
We definitely know from the equation (9), (12) and (13) have negative Eigne values. Consider equation (10) which implies, $X_2 = A < 0$ iff $(\mu_1 + \mu_2 + \beta_2) > \beta_1 S^*$. Equation (11) implies $X_3 < 0$ iff $1/\mu_h > R_0$. Which complete the proof Hence we say that local stability analysis at endemic equilibrium of the model (1) is asymptotically stable.

Global Stability Analysis

In study of the mathematical epidemiology global stability at equilibria considered the key problem. Fortunately for this we have a Lyapunov function to check stability analysis is a power full tool of the autonomous system. Thus to find here the global stability of mathematical model (1), we using the concept of Lyapunov function, which used by many authors [28,29]. Now we elaborate the global stability of model (1) at disease free equilibrium. For this we need to construct the Lyapunov function. Thus following are the stability results.

Global Stability Analysis at Disease Free Equilibrium

Theorem: *For R_0 1, the model* (1) *is said to be globally asymptotically stable on the point disease free equilibrium, if $S = S_0$ and unstable for R_0 >1.*

Proof: For global stability analysis at disease free equilibrium of the model (1), we need to define the *Lyapunov function*, consider,

$$L(S, I, R, S_c, I_c) = \frac{1}{2}[S - S^0 + I]^2 + (S_c - S_c^0).$$

Clearly the above define function is L(S, I, R, S_c, I_c) > 0 and also (S(0); I_h(0), R(0), S_c(0), I_c(0)) = 0. Now differentiate the function L(S, I, R, S_c, I_c) with respect to *t*, we get,

$$\frac{d}{dt}L(S, I, R, S_c, I_c) = [S - S^0 + I_h][\frac{d}{dt}S + \frac{d}{dt}I] + \frac{d}{dt}S_c.$$

Taking values from model (1), the above equation becomes,

$$\frac{d}{dt}L(S, I, R, S_c, I_c) = [S - S_h^0 + I]\{\mu_p - \mu_h S - (\mu_1 + \mu_2 + \beta_2)I\} + (\mu_c - (\beta_4 I_c + \mu_{sc})S_c).$$

Hence dL/dt = 0 if and only if S = S_0, I = I_0, R = R_o, S_c = S_c^o and I_c = I_c^o and dL/dt < 0 iff μc < k for S > S_0 also $k = \mu_3 + \frac{2\mu_{sc(\mu_3+\mu_4)}}{\beta_4} - \mu_c$ Hence disease free equilibrium is globally asymptotically stable which complete the proof.

Global Stability analysis At Endemic Equilibrium

Theorem: *For R_0 > 1, the endemic equilibrium of the model* (1) *is globally asymptotically stable, if $S = S^*$, $I = I^*$, $R = R^*$, $S_c = S_c^*$ and $I_c = I_c^*$ and unstable, if R_0 is less then unity.*

Proof: Now we discuss the global stability analysis of model (1) at endemic equilibrium. For that purpose we define the following *Lyapunov function,*

$$W(S^\star, I^\star, R^\star, S_c^\star, I_c^\star) = \frac{1}{2}(S_c - S_c^\star)^2 + \frac{1}{2}(I_c - I_c^\star)^2.$$

It clear that W(S*, I*, R*,S^*_c,I^*_c) > 0 and W (S*, I* , R*,S^*_c , I^*_c = 0 iff Sc = S^*_c and I_c = I^*_c . Differentiating with respect to "t" we have,

$$\frac{d}{dt}W(S^\star, I^\star, R^\star, S_c^\star, I_c^\star) = (S_c - S_c^\star)(\frac{d}{dt}(S_c)) + (I_c - I_c^\star)(\frac{d}{dt}(I_c)).$$

Using values from model (1), we get,

$$\begin{aligned}\frac{d}{dt}W(S^\star, I^\star, R^\star, S_c^\star, I_c^\star) &= (S_c - S_c^\star)(\mu_c - \beta_4 S_c I_c - \mu_{sc} S_c) + (I_c - I_c^\star)(\beta_4 S_c I_c - (\mu_3 + \mu_4)I_c),\\ \frac{d}{dt}W(S^\star, I^\star, R^\star, S_c^\star, I_c^\star) &= -(S_c - S_c^\star)(k - \mu_c).\end{aligned}$$

Hence dW/dt = 0 if and only if S_c = S_c^* and I_c = I_c^* and also dW/dt < 0, iff k > μ_c which complete the proof. Hence the endemic equilibria of the model (1) is globally asymptotically stable.

Optimal Control Problem of the Model

To prevent MERS-corona virus in the Arabian community we need to draw special methodology for its control, that is, (we define some control variable) to reduce infection from individuals. For that purpose optimal control is one of the powerful tool considered in Mathematics to check the control of some infectious diseases. Optimal control is used for the development and improvement of the control strategy [30]. In normal population we use control variable tool to minimize MERS-Corona disease.

We use
$M_1 : u_1(t)$ a mask to cover sensitive part of body and
$M_2 : u_2(t)$ using medication or antivirus in advance which prevent the spread of disease.

In model (1) we have five variables with names S Susceptible individuals, I Infected individuals, R Recover individuals, S_c Susceptible camels and I_c Infected camels respectively. For minimization of objective functional an optimal control with the help of two control variables are define by the following.

$$L(u_1, u_2) = \int [(A_1 S(t) + A_2 I(t) + A_3 R(t) + A_4 S_c(t) + A_5 I_c(t)) + \frac{1}{2}(w_1 u_1^2(t) + w_2 u_2^2(t))]dt. \quad (14)$$

Subjected to M_1 and M_2 we have the following system of equations,

$$\begin{aligned}
\frac{dS(t)}{dt} &= \mu_p - \beta_1 SI - \mu_h S - \beta_3 SI_c + u_1 I(t) + u_2 R(t), \\
\frac{dI(t)}{dt} &= \beta_1 SI + \beta_3 SI_c - (\mu_1 + \mu_2)I - \beta_2 I + u_1 I(t), \\
\frac{dR(t)}{dt} &= \beta_2 I - \mu R, \qquad (15) \\
\frac{dS_c(t)}{dt} &= \mu_c - \beta_4 S_c I_c - \chi S_c - u_1 S(t), \\
\frac{dI_c(t)}{dt} &= \beta_4 S_c I_c - (\mu_3 + \mu_4) I_c.
\end{aligned}$$

Considered with initial conditions below,

$$S(0) \geq 0, I(0) \geq 0, R(0) \geq 0, S_c(0) \geq 0, I_c(0) \geq 0.$$

In above equation (15) the terms A_1 represent susceptible individuals, A_2 represent infected individuals, A_3 assign for recovered individuals, A_4 represent susceptible camels, and A_5 show infected camels. Also $\frac{1}{2} w_1 u_1^2(t)$ used to wear mask to cover sensitive part of body parts and $\frac{1}{2} w_2 u_1^2(t)$ represent the medication or antivirus associated with the proposed problem. While in first we need to derive the control function, given by below,

$$L(u_1^\star, u_2^\star) = min\{L(u_1, u_2), u_1, u_2 \varepsilon W\}. \qquad (16)$$

Subjected to the system (15) also the control set variables for the proposed model is as, $W = \{(u_1, u_2) / u_i(t)$ is Lebesgue measurable on $[0, 1], 0 < u_i(t), i = 1, 2\}$

Existence of the optimal Control Problem

Now consider the control system (15) in initial time $t = 0$ to show the existence of the optimal control problem. The bounded Lebesgue measurable control, positive initial condition, and positive bounded to the state system exist [31]. We find optimal control solution of problem for that consider equation (14) and (15). For that purpose first we define the Lagrangian for optimal control problem as,

$$L(S, I, R, S_c, I_c, u_1, u_2) = A_1 S(t) + A_2 I(t) + A_3 R(t) + A_4 S_c(t), + A_5 I_c(t) + \frac{1}{2}(w_1 u_1^2 + w_2 u_2^2).$$

For finding the minimal value of the Lagrangian we define the Hamiltonian "H" as.

$$H = L(S(t), I(t), R(t), S_c(t), I_c(t), u_1(t), u_2(t)) + \lambda_1 \frac{dS}{dt} + \lambda_2 \frac{dI}{dt} + \lambda_3 \frac{dR}{dt} + \lambda_4 \frac{dS_c}{dt} + \lambda_5 \frac{dI_c}{dt}. \quad (17)$$

For existence of the proposed model we have the following result,

Theorem : *For existence of optimal control we take* $u^* = u_1^*, u_2^* \, \varepsilon W$ *such that,* $L(u_1^*, u_2^*) = minL(u_1^*, u_2^*)$.
Subjected to the initial condition of the control system (17).

Proof: To prove optimal control existence using in we define positive control variables and state variables. For minimizing the case define above the convexity required for the objective functional in equations (14) $u_1(t)$ and $u_2(t)$ are satisfied [5]. The set of control variables u_1, $u_2 \, \varepsilon W$ and by definition it closed and also convex. Here optimal control system is bounded which show the compactness and fulfil the existence of the proposed model and optimal control for further to integrand on objective functional (14),

$A_1S(t) + A_2I(t) + A_3R(t) + A_4S_c(t) + A_5I_c(t) + ½ (w_1u^2{}_1(t) + w_2u^2{}_2(t))$.

Taking the convex in optimal control, that is, set W that implies the ensure of optimal control (u_1^*, u_2^*)
to minimize (15). Now for optimal control problem we need to find optimal control solution for purposed model for this we use Maximum principle of Pontryagin,s to the Hamiltonian [32].

$H(y, p(y), u(y), \lambda(y)) = f(y, p(y), u(y) + \lambda(g(p(y), u(y)))$

If (p^*, u_1^*, u_2^*) we will considered the optimal control solution for the required proposed optimal control problem then obviously there a non-trivial vector exist,

$\lambda(y) = (\lambda_1(y), \lambda_2(y), \lambda_3(y)\ldots\ldots\ldots\ldots, \lambda_n(y))$.

Such that y taking for time, that is, $y = t$

$$\begin{aligned} \frac{dx}{dy} &= \partial H(y, x(y), u(y), \lambda(y)) \\ 0 &= \frac{\partial H(y, p(t), u(y), u(y), \lambda(y)}{\partial u} \\ \lambda(y)' &= \frac{\partial H(y, p(y), u(y), \lambda(y)}{\partial x}. \end{aligned} \quad (19)$$

Now on Hamiltonian equation we apply the necessary condition and then we processed as.

Theorem: Suppose that $S^*, I^*, R^*, S_c^*, I_c^*$ are the optimal state solution regarding to optimal control variables u_1^*, u_2^* for the optimal problem (14) also (15). Then the adjoint variables will exist there $\lambda_1(y), \lambda_2(y), \lambda_3(y), \lambda_4(y), \lambda_5(y)$ are satisfied.

$$\begin{aligned} \lambda_1'(y) &= -A_1 + (\lambda_1 + \lambda_2)\mu_h, \\ \lambda_2'(y) &= -A_2 + \lambda_1(\beta_1 S + \beta_3 S) - \lambda_2(\beta_1 S - (\mu_1 + \mu_2) - \beta_2), \\ \lambda_3'(y) &= -A_3\lambda_3\mu, \\ \lambda_4'(y) &= -A_4 + \lambda_4(\beta_4 I_c + \chi) - \lambda_5\beta_4 I_c, \\ \lambda_5'(y) &= -A_5 + \lambda_4\beta_4 S_c - \lambda_5(\beta_4 S_c - (\mu_3 + \mu_4). \end{aligned} \quad (20)$$

with the transversality conditions (Boundary conditions). $\lambda_i(y) = 0$, for i = 1, 2, 3.
Further more the optimal control variables u_1^* and u_2^* are as.

$$u_1^* = max\{min\{\frac{\lambda_4 S(t) - (\lambda_1 + \lambda_2)I(t)}{w_1}, 1\}, 0\} \quad (21)$$

$$u_2^* = max\{min\{\frac{-\lambda_1 R(t)}{w_2}, 1\}, 0\} \quad (22)$$

Proof: Now to find the adjoint equation (15) for transversality conditions (21), considered the Hamiltonian (17) by assigning $S = S^*$, $I = I^*$, $R = R^*$, $S_c = S_c^*$ and Ic = I_c^* ; then differentiate Hamiltonian equation with respect to S, I, R, S_c and I_c. Then we obtain the desired adjoint equation (24). We find u_1^* and u_2^*. Now to differentiate the Hamiltonian with respect to u_1 and u_2 we solve $\partial H/\partial u_1 = 0$ and $\partial H/\partial u_2 = 0$ on the interior on the control set we use optimality condition. Finally we use a property of the control space W and we get equation (23) and (24) which complete the required proof. Now again we call equations (23) and (24) from the optimal control u* the characterization of the optimal control. We obtained the State variables and also optimal control variables by the solving optimality system which contain state variables (14) and adjoint system (15) by boundary condition (23) and (24). Putting the values of u_1^* and u_2^* in the control system (15) we get the following.

$$\begin{aligned}
\frac{dS^*(t)}{dt} &= \mu_p - \beta_1 SI - \mu_h S - \beta_3 SI_c + max\{min\{\frac{\lambda_4 S(t) - (\lambda_1 + \lambda_2)}{w_1}, 1\}, 0\}I(t) \\
&+ max\{min\{\frac{-\lambda_1 R(t)}{w_2}, 1\}, 0\}R(t), \\
\frac{dI^*(t)}{dt} &= \beta_1 SI + \beta_3 SI_c - (\mu_1 + \mu_2)I - \beta_2 I + max\{min\{\frac{\lambda_4 S(t) - (\lambda_1 + \lambda_2)}{w_1}, 1\}, 0\}I_h(t), \\
\frac{dR^*(t)}{dt} &= \beta_2 I - \mu R, \\
\frac{dS_c^*(t)}{dt} &= \mu_c - \beta_4 S_c I_c - \chi S_c - max\{min\{\frac{\lambda_4 S(t) - (\lambda_1 + \lambda_2)}{w_1}, 1\}, 0\}S(t), \\
\frac{dI_c^*(t)}{dt} &= \beta_4 S_c I_c - (\mu_3 + \mu_4)I_c.
\end{aligned}$$

With H*at $(t, S^*, I^*, R^*, S_c^*, I_c^*, u_1^*, u_2^*, \lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5)$ such that,

$$\begin{aligned}
H &= A_1 S^* + A_2 I^* + A_3 R^* + A_4 S_c^* \\
&+ A_5 I_c^* + \frac{1}{2} w_1 [max\{min\{\frac{\lambda_4 S(t) - (\lambda_1 + \lambda_2)}{w_1}, 1\}, 0\}I(t)]^2 \\
&+ \frac{1}{2}[max\{min\{\frac{-\lambda_1 R(t)}{w_2}, 1\}, 0\}]^2 \\
&+ \lambda_1 \frac{dS^*(t)}{dt} + \lambda_2 \frac{dI^*(t)}{dt} + \lambda_3 \frac{dR^*(t)}{dt} + \lambda_4 \frac{dS_c^*(t)}{dt} + \lambda_5 \frac{dI_c^*(t)}{dt}. \qquad (23)
\end{aligned}$$

Now we solve numerically the optimal control system (24) for the sack of optimal control.

Numerical Simulation result applied to the control problem

In this subsection of the article, now we solve model (1) by numerical simulation through Runge-kutta, that is, RK_4 method. Runge-Kutta method is used in to solve different wide range of differential equation problems [33-35]. The different parameters values provide different numerical results are given in (Figures 1, 2, 3, 4 and 5). Parameters description, notation and their values used in Table at section 10. In our problem we have two types of population, human population and vector camel population. In the simulation the Figure 1 represent the graph of susceptible individuals, where recovery is very fast with vaccination while without vaccination we see that the graph is increasing, showing bed hygienic condition of the susceptible individuals. Figure 2 show the graph of infected individuals of MERS corona virus where disease spread rapidly without control and in this way it reach to uncontrollable level, but by applying vaccine to this class of individuals recovery is very sharp and quick. Figure 3 describe the graph of recover individuals, which rapidly recovered by using vaccine or control. By using vaccine the individuals recovery line is going straight, mean recovery is very fast as compared to non vaccination graph Now in vector camel population Figure 4 represent the graph of susceptible camel which initially high but vaccine slow down its intensity and brought the graph to zero as compared to the non vaccinated its going high and not touch the zero. In Figure 5 which show the graph of infected camel going maximum and finally in this level no recovery is possible but when we apply vaccine or control strategy we observe that the recovery is very fast. In our article the mission is to applying the optimal control to reduce the number of infection individuals and increase the number of normal individuals. All the values in the Table given in section 14 are fixed. Graph of the Figures 1, 2, 3, 4 and 5 in the simulation are shown with and without control. It is clear from simulation that with vaccination the status is very effective and the recovery is very much faster while in case of without vaccination the recovery in very slow or we say without vaccine the recovery take very large time.

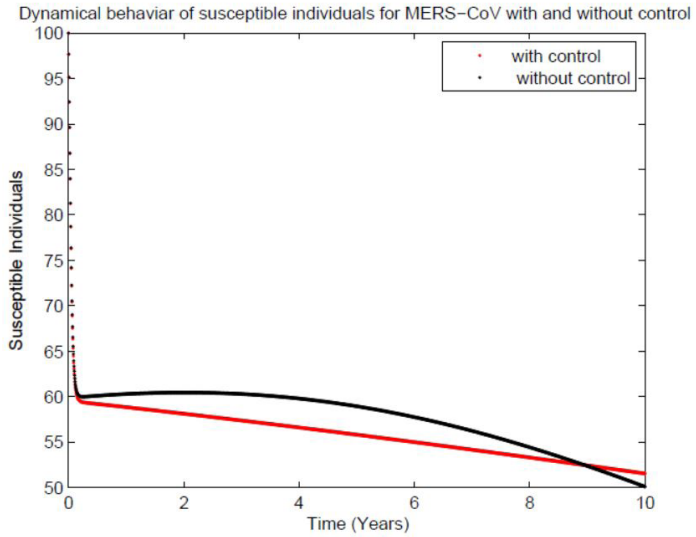

Figure 1: The plot shows the population of susceptible individuals with and without control for MERS-CoV.

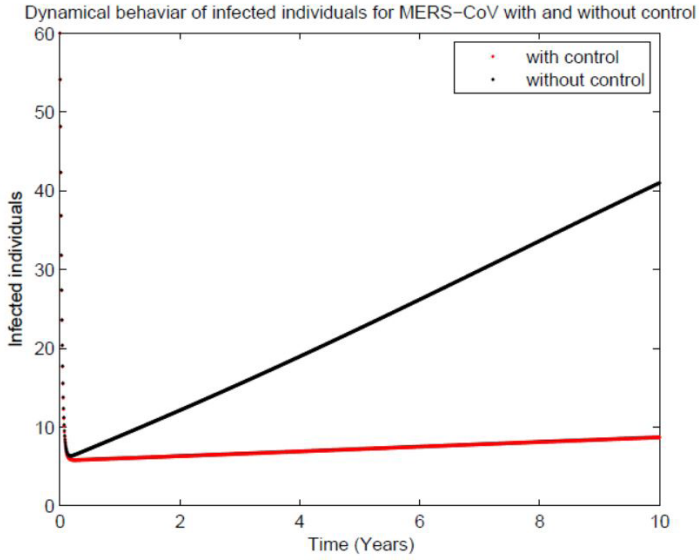

Figure 2: The plot shows the population of infected individuals with and without control for MERS-CoV.

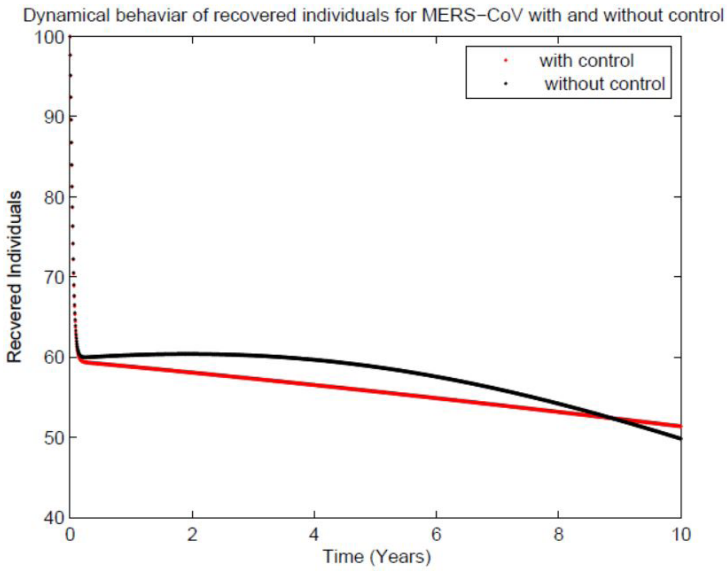

Figure 3: The plot shows the population of recovered individuals with and without control for MERS-CoV.

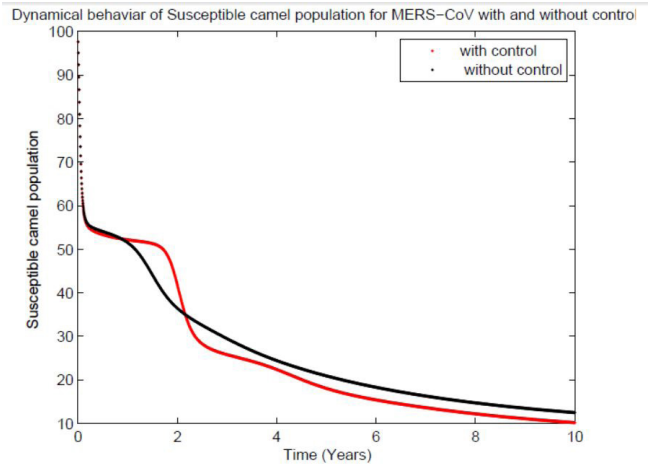

Figure 4: The plot shows the population of susceptible camel with and without control for MERS-CoV.

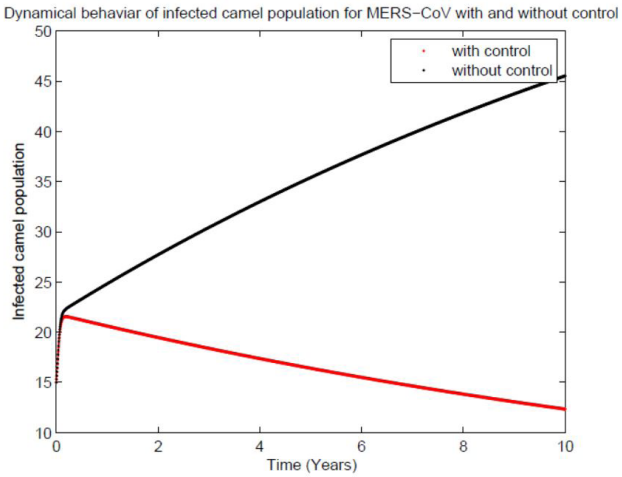

Figure 5: The Plot shows population of infected camel with and without control for MERS-CoV.

Notation	Description of Parameter	Value
S	The Susceptible individual population	50-100
I	The Infected individual population	00-60
R	The Recovered individual population	40-100
S_c	The Susceptible camel population	10-100
I_c	The Infected camel population	10-50
λ	New birth rate in susceptible individuals	0.6321
β_1	The Transmission rate from susceptible to exposed individuals	0.2877
β_2	The Transmission rate from exposed to infected individuals	0.7613
β_3	The Transmission rate from infected to recover individuals	0.4389
β_4	The Individuals get wild animals infection from susceptible to exposed	0.1234
β_5	The Individuals get wild animals infection from susceptible to infected	0.2431
β_6	The Individuals get domestic animals infection from susceptible to exposed	0.4
β_7	The Individuals get domestic animals infection from susceptible to infected	0.3
μ	The Natural death rate of susceptible individuals	0.9704
μ_1	The Natural death rate of exposed individuals	0.0432
μ_2	The Infectious death rate of exposed individuals	0.2006

Notation	Description of Parameter	Value
μ_3	The Natural death rate of infected individuals	0.0656
μ_4	The Infectious death rate of infected individuals	0.9764
μ_5	The Natural death rate of recover individuals	0.6704
v	without vaccination or controls	0
u_1	Using mask to cover body	1.99
u_2	Using medication or Anti virus	3.5

Table 1: Description of Parameter, notation and their values

Conclusion about the Proposed Model

Here a mathematical epidemic model for both, human population and vector camel population of MERS corona virus is considered. In the model the transmission of infection is spread from infected camel population to susceptible individual population, which specially effected the owner, shepherd and cherisher individuals and further they spread infection in human community. In the article we proceeded as, after the introduction and related literature in the first, we calculated the basic key value, that is, reproductive number R0, by the using of next generation approach. Then we derived all the endemic equilibrium points of the concern model. Further in presence of reproductive number we shown the stability analysis, that is, local stability analysis and global equilibria were stable. While for global stability analysis we developed the technique of Lyapunov function and discussed its stability. Finally we obtained numerical solution of mathematical model by the use of Runge-Kutta method of order 4four tool and presented the results from Figure 1 to 5 with and without

Conflict of Interest

There is no conflict of interest regarding this paper.

Authors contribution

All authors equally contributed this paper.

Acknowledgments

All authors read and approved the final version.

References

1. WHO (2017) WHO MERS-CoV Global Summary and Assessment of Risk.

2. Pontryagin L, Boityanskii V, Gramkrelidze R, Mischenko E (1962) The mathematical theory of optimal processes (2nd edn) Wiley, Hoboken, NJ.

3. WHO (2014) Infection prevention and control during health care for probable or confirmed cases of novel coronavirus (nCoV) infection. World health Org.

4. Who Mers-Cov Res G (2013) State of knowledge and data gaps of Middle East res- piratory syndrome corona virus (MERS-COV) in humans. PLoS Curr 5.

5. Huffington Post (2014) Saudi Arabia finds another 18 MERS cases as disease spreads. The Huffington Post Reuters.

6. Omar Abu Arqub (2017) Solution for time-fractional Tricomi and Keldysh equation of Dirichlet function types in Hilbert space. Numer Methods Partial differ Equation 34: 1759-80.

7. Tahir M, Syed Inayat Ali Shah, Gul Zaman, Sher Muhammad (2018) Ebola virus epidemic disease its modeling and stability analysis required abstain strategies Cogent Biol 4: 1488511.

8. A Zumla, DS Hui, S Perlman (2015) Middle East respiratory syndrome. Lancet, 386: 995-1007.

9. Omar Abu Arqub (2016) Approximate solution of DASs with nonclassical boundery conditions using novel reproducing kernel algorithem, Fundamenta Informaticae 146: 231-54.

10. Magombedze G, Mukandavire Z, Chiyaka C, Musuka G (2009) Optimal control of a sex structureed HIV/AIDS model with condom use. Math Model Anal 14: 483-94.

11. Messac A, Mattson C (2004) Normal consraint method with guarantee of even representation of complete Pareto frontier AIAA J 42: 2101-11.

12. Pascoletti A, Serafine P (1984) Scalarizing vector optimization problem. J Optim Theory Appl 42: 499-524.

13. Omar Abu Arqub (2017) Fitted reproducing kernal Hilbert space method for the solution of some certain classes of time fractional partial dierential equation subject to initial and Neumann boundary conditions. Comput Math Appl 73: 1243-61.

14. Omar Abu Arqub (2018) Numerical solution for the Robin time-fractional partial dierential equationof heat and fluid flows based on the reproducing kernel algorithem, Int J Numer Methods for heat and fluid flow 28: 828-56.

15. A Mwasa, JM Tchuenche, Mathematical analysis of a cholera model with public health interventions. Bio Systems 3: 190-200.

16. Doucleef M (2012) Scientists Go Deep On Genes of SARS-Like Virus. NPR Associated Press.

17. Ibrahim Oruko (2016) Anger in the north as mysterious disease wipe out camels. The Star Kenya.

18. Gallagher James (2014) Camel infection 'led to Mers death' BBC News.

19. Omar Abu Arqub, Al-Smadhi M, Shawagfeh N (2013) Solving Fredholm integro-differential equations using reproducing kernel Hilbert space method. Appl Math Comput 219: 8938-48.

20. Ken Bet (2016) Herders counting losses as mysterious disease kills over 500 camels in Marsabit. Daily Nation.

21. Azhar EI, El-Kafrawy SA, Farraj SA, Hassan AM, Al-Saeed MS, et al. (2014) Evidence for Camel-to-Human Transmission of MERS Coronavirus' NEJM 370: 2499-505.

22. Declan Butler (2013) Receptor for new coronavirus found. Nature 495.

23. Omar Abu Arqub (2016) The reproducing kernal algorithem for handling differential algebraic system of ordinary differential equation, Math Methods Appl Sci 39: 4549-62.

24. Zaki AM, van Boheemen S, Bestebroer TM, Osterhaus AD, Fouchier RA (2012) Isolation of a novel coronavirus from a man with pneumonia in Saudi Arabia. N Engl J Med 367: 1814-20.

25. McKenna Maryn (2015) MERS Cases Increasing in Saudi Arabia, And The Hajj Is Coming. National Geographic.

26. Daniel KW Chu, Leo LM Poon, Mokhtar M Gomaa, Mahmoud M Shehata, Ranawaka APM P, et al. (2014) MERS Coronaviruses in Dromedary Camels, Egypt. Emerg Infect Dis 20.

27. Omar Abu Arqub, Al-Smadhi M (2014) Numerical algorithem for solving two point, second-order periodic boundary value problems for mixed integro-differential equations, Appl Math Comput 243: 911-22.

28. S Thornley, C Bullen, M Roberts (2008) Hepatitis B in a high prevalence New Zealand Population a mathematical model applied to infection control policy. J Theor Biol, 254: 599-603.

29. MA Khan, S Islam, M Arif, Z. ul Haq (2013) Transmission Model of Hepatitis B Virus with the Migration Effect. BioMed Res Int 150681.

30. IK Mortan, LS Nancy (2000) Dynamic optimization: The calculus of variation and optimal control in Economocs and Management. Els Sci Netherland 218-34.

31. G Birkho, GC Rota (1989) ordinary differential equations, (4th edn). JohnWiley and Sons, N Y.

32. L Jodara, FJ Santonjaa, G Gonzalez-Parra (2008) Modeling dynamics of infant obesity in the region of Valencia, Spain. Comp Math Appl 56: 679-89.

33. RM Anderson, RM May (1991) Infectious Diseases of Humans. Dyn and Cont, Oxford Univ Press.

34. OD Makinde (2007) Adomian decomposition approach to a SIR epidemic model with constant vaccination strategy. Appl Math Comput 184-848.

35. Y Khan (2009) An effective modification of the Laplace decomposition method for nonlinear Equations, Int J Sci 10: 1373-76.