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Optimization of Physics-Mechanical Parameters of Hardening Complex Metal Coatings: Regression-Tensor Approach

Rusanov VA[1*], Agafonov SV[2], Daneev RA[3] and Syrovatskaya EV[4]

[1]*Doctor of Physical and Mathematical Sciences, Chief Researcher for Matrosov Institute for System Dynamics and Control Theory of the Siberian Branch of the Russian Academy of Sciences (ISDCT SB RAS), Irkutsk, Russia*

[2]*Doctor of Technical Sciences, Professor for Irkutsk State Agricultural Academy, Irkutsk, Russia*

[3]*Doctor of Technical Sciences, Teacher for East-Siberian Institute of MIA RF, Irkutsk, Russia*

[4]*Researcher for Irkutsk State Transport University, Irkutsk, Russia*

[*]**Corresponding author:** Rusanov VA, Doctor of Physical and Mathematical Sciences, Chief Researcher for Matrosov Institute for System Dynamics and Control Theory of the Siberian Branch of the Russian Academy of Sciences, ISDCT SB RAS, 134, Lermontov st., 664033, Irkutsk, Russia, Tel: 89086669007, E-mail: v.rusanov@mail.ru

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Abstract

A nonlinear multidimensional regression-tensor model is constructed and investigated to the end of grounding (necessary and sufficient conditions are implied) of an optimal multifactor physics-chemical process of hardening metal coatings. A robust-adaptive strategy of rational forming the goal functional of physics-mechanical quality of metal working is proposed. The results obtained may form a methodological ground in constructing the systems of computer-aided design, technologies of hardening surfaces of complex composite fabricated metal products of the basis of complex tribological tests.

Keywords: Tribological tests; Regression-Tensor Model; Hardening Metal Coatings

Introduction

Development of methods of hardening the working surfaces of cutting machines presumes complex *physics-chemical processes* (PCP). So, essential still are the issues of formalization/processing the respective mathematical models. In the given context, regression models (linear ones, nonlinear ones, including matrix ones, where the important class of systems is represented by regression-tensor systems are in demand [1-7]. On the one hand, from the viewpoint of properties, these systems are quite close to polynomial ones, which presume a rather detailed analytical description on the basis of (i) tensor calculus, (ii) strong differentiability of vector mappings and (iii) the theory of extreme problems [2,7,8]. On the other hand, these systems acquire an important role in nonlinear modeling multifactor tribological properties of synthesized metal coatings, in particular, in prognostic description of surface nano-dimensional structures [9,10].

Below considered are the problems stated in the conclusions of paper, while the objective is not the formal precision conclusions but rather the clarity of the conceptions in development of tribological problems [5,11]. Hence the issue of forming the functional of metal coating physics-mechanical properties for the mode of hardening is solved in this context. Determined are the strong analytical interpretations of the interconnected conditions, which define an optimal mode of the PCP given by nonlinear constraints and providing for adequacy of the PCP model to the data of tribological tests [12,13]. So, solved is the problem of multicriterial identification (by the least squares method (LSM)) of coordinates of covariant tensors in the PCP equation as a multidimensional regression with a minimum tensor norm.

Principal provisions and statement of the problem of regression-tensor modeling an optimum PCP

Let R be a field of real numbers, R^n be an n-dimensional vector space over R with the Euclidean norm $||.||_{R^n}$, $col(y_1,\dots,y_n)\in R^n$ be a vector column with the elements $y_1,\dots,y_n \in R$, and let $M_{n,m}(R)$ be a space of all $n{\times}m$-matrices with the elements from R. Next, let us denote by T_m^k the space of all covariant tensors of k-th valence (of real poly-linear forms $f^{k,m}: R_1^m\times\dots\times R_k^m \to R$) with the tensor norm $\| f^{k,m} \|_T := \left(\sum t_{i...j}^2\right)^{1/2}$, where $t_{i...j}$ are coefficients (coordinates of tensor $f^{k,m}$, whose values are given with respect to the standard (natural) orthonormalized basis in the Euclidean space R^m [14].

Let $v \in R^m$ be a vector of varied physics-chemical predictors of regression of PCP with a fixed beginning in $\omega \in R_m$ (support mode of hardening), $w(\omega + v) \in R^n$ is a vector of qualitative indicators of PCP [2]. As far as the given problem statement is concerned, consider a multidimensional nonlinear "input-output" system described by the following vector-tensor k-valent equation of multifactor regression

$$w(\omega+v) = col\left(\sum_{j=0,\dots,k} f_1^{j,m}(v,\dots,v),\dots, \sum_{j=0,\dots,k} f_n^{j,m}(v,\dots,v)\right) + \varepsilon(\omega,v), \quad (1)$$

where $f_1^{j,m} \in T_m^j$, vector function $\varepsilon(\omega,.) : R^m \to R^n$ belongs to the class

$$\| \varepsilon(w,v) \|_{R^n} = o((v_1^2 + \dots + v_m^2)^{k/2}) \quad (1^*)$$

Where $v = col(v_1,\dots,v_m), f_i^{0,m} (1 \le i \le n)$ are invariant tensors of zero valence (tribological quality indicators of the scrutinized physics-chemical process in the supportmode $\omega \in R^m$ of its physics-chemical predictors) [7,11].

Remark 1: The description of PCP in terms of the regression system (1) is adequate on account of Proposition 2 related to continuous dependence of the solution of the PCP differential equation on the initial-boundary conditions and on the parameters [5,8,15].

The problem of an a posteriori regression-tensor modeling of an optimum PCP has been stated and investigated in detail in for a bivalent model (1). Furthermore, in the analytical solutions of three positions of the problem have been obtained [5]:

1) for a fixed index k, a given predictor $\omega \in R^m$ and $v \subset R^m$, i.e. an open neighborhood of vector ω defined are the analytical conditions, under which the vector function $w(.) : V \to R^n$ of PCP quality indicators satisfies system (1);

2) constructed is an algorithm of identification of coordinates of symmetric tensors $f_1^{j,m}, 1 \le i \le n, 0 \le i \le k = 2$ in the mathematical model of PCP (1) on the basis of bicriterial LSM-problem (2) (parametric LSM-identification of a multidimensional regression-tensor system (1) with the minimal tensor norm) [16]:

$$\begin{aligned} &\min\Big(\sum_{l=1,\dots,q} (\| w_{(l)} - col(\sum_{j=0,\dots,k} f_1^{j,m}\left(v_{(l)},\dots,v_{(l)}\right),\dots, \sum_{j=0,\dots,k} f_n^{jm}\left(v_{(l)},\dots,v_{(l)}\right)) \|_{R^n})^2\Big)^{1/2}, \\ &\min\Big(\sum_{i=1,\dots,n}\sum_{j=0,\dots,k} \| f_1^{jm} \|_T^2\Big)^{1/2}; \end{aligned} \quad (2)$$

where $w_{(l)} \in R^n, v_{(l)} \in R^m, 1 \le l \le q$ are vectors of experimental factor predictors of PCP ($w_{(l)}$ is a "reaction" to the "variation" $v_{(l)}$ with respect to the "mode" $\omega \in R^m$, furthermore, $\| v_{(l)} \|_{R^m} < 1$, what is determined by condition (1^*)), q is the number of tribological experiments with PCP; under such a present problem statement an approach proposed in may be applied [17,18].

3) under the given predicting vector $\omega \in R^m$ and $\varepsilon(\omega,v) \equiv 0$ for the case of bivalent regression-tensor model (1) we have obtained an analytical solution corresponding to "v-optimization" of the quadratic function of varied predicting factors for the scrutinized PCP with respect to its support mode ω (a "support vector" of predicting factors):

$$\max\{F(v) : v \in R^m\}, \quad (3)$$

$$F(v) := r_1 w_1(\omega+v) + \dots + r_n w_n(\omega+v),$$

where vector function $v \to col(w_1(\omega+v), \dots, w_n(\omega+v)) = w(\omega+v) \in R^n$ has the coordinate representation corresponding to the identifies model (1)-(2); $r_i > 0$ are the weighting coefficients, which reflect a relative priority of some of tribological characteristics w_i, $1 \le i \le n$ of PCP physics-mechanical properties.

Problem statement (on the basis of the conclusion from) [5]. It is required to determine the *necessary* conditions in the solution of problem (3) when $k = 3$ (finding the stationary points in (3) for the 3-valent model (1)), and complement this determination with finding *sufficient* conditions of "v-optimization", i.e. provision for the "elliptic character" of functional F critical points at the expense of dependence of spectral characteristics of its Hessian on variations of vector $r := \mathrm{col}(r_1, \dots, r_n)$ with respect to some "initial" positions $r_0 \in R^n$.

Constructing an optimal PCP

Consider the case of equations of multidimensional regression with the tensor structure of valence $k = 3$; solving problem (2) when

$k = 3$ represents a trivial modification of the proof of Proposition 3 [5]. Under such a problem statement, the system of equations (1) may be represented in the vector-matrix-tensor form

$$w(\omega+v) = c + Av + col(v^1 B_1 v + f_1^{3,m}(v,\ldots,v), \ \ldots, \ \ldots, v^1 B_n v + f_n^{3,m}(v,\ldots,v)) + \varepsilon(\omega, v), \tag{4}$$

$$c \in R^n, A \in M_{n,m}(R), B_i \in M_{m,m}(R), i = 1,\ldots,n$$

(in this case, we assume that each B_i is an upper triangular matrix), from now on, the upper prime index ` denotes the operation of transposition of either a vector or a matrix; the vector function $\varepsilon(\omega,\cdot): R^m \to R^n$ satisfies (due to condition (1^*) and Proposition 2 the following analytical estimate [5].

$$\| \varepsilon(w,v) \|_{R^n} = o((v_1^2 + \ \ldots \ + v_m^2)^{3/2}).$$

When $k = 3$, functional F is twice continuously differentiable, what guarantees the equality of mixed derivatives∂ $\partial^2 F(v_1,\ldots,v_m)/\partial v_g \partial v_p = \partial^2 F(v_1,\ldots,v_m)/\partial v_p \partial v_g, g, p = 1,\ldots,m,$ therefore, the principal result of solving problem (3) (due to Theorem 3 and Theorem 7.2.5. for the 3-valent model (4) presumes the following proposition [8,14].

Proposition 1: *Let $B_i^* := (B_i + B`_i) \in M_{m,m}(R)$, $1 \le i \le n$, where each B_i is a matrix of system (4) and, furthermore, consider the following vector function*

$$\Phi(v) := (r_1 + B_1^* \ \ldots \ + r_n B_n^*)^{-1}(A' + + [\nabla_v f_1^{3,m}(v,\ldots,v),\ldots,\nabla_v f_n^{3,m}(v,\ldots,v)])r.$$

Hence the stationary points $v^* \in R_m$ of problem (3) is solutions of equation

$$v^* + \Phi(v^*) = 0, \tag{5}$$

in this case, the sufficient condition of the statement that point v^* of the space of predicting factors provides for "maximum quality of PCP" of the form

$$max\{F(v): v \in R^m\}, F(v) = r' w(\omega+v)$$

represents the following requirement: v^* as a critical point of functional $F(v)$ must be of special elliptic type. This is precisely the same as the statement that

$$\det\, [b_{ij}]_p < 0, \, p=1,\ldots,m, \tag{6}$$

where $[b_{ij}]_p \in M_{PP}(R), p = 1,\ldots,m,$ are main submatrices of Hessian $G(r)$ at point $v^* \in R_m$

$$G(r) = (r_1(B_1^* + [\partial^2 f_1^{3,m}(v,\ldots,v)/\partial v_g \partial v_p \,|\, v^*]) + \ldots\ldots + r_n(B_n^* + [\partial^2 f_n^{3,m}(v,\ldots,v)/\partial v_g \partial v_p \,|\, v^*]) \in M_{m,m}(R),$$

or similarly to state that the characteristic numbers λ_p of matrix $G(r)$ satisfy the condition

$$\lambda_p < 0, p = 1,\ldots,m. \tag{7}$$

Corollary 1. *When $k = 2$, Hessian $G(r)$ of functional F is*

$$G \ r \ = r \ \ ^* + \ \ldots \ + r \ \ ^*,$$

furthermore, when rank $G(r) = m$, the solution of equation (5) is unique and has the form

$$v^* = -G^{-1}(r) A' r.$$

Obviously, (5) represents an intersection of m quadrics so, if conditions (6) (or, similarly, (7)) are not satisfied, then the critical point (/points) (5) is hyperbolic (saddle) one. Therefore, existence of a saddle point guaranteed by the replacement of < in (6) or (7) with > at least in one (not in all) inequalities (see e.g. (16) [5]). Replacement of inequality < with the reflexive ≤ induces in v^* the structure of stationary parabolic point of functional $F(\cdot)$; in this case, rank $G(r) < m$, consequently, some additional analysis of (5) is needed. In such a situation, some parametric correction of functional (3) is required in order ensure the elliptic character of (6).

It is clear that coordinate adjustment of vector r is one of the factors, which influence the geometry of $F(\cdot)$ at the critical point v^*. This determines the statement of the problem of "adaptive correction" $r \to r' w(\omega+v)$ for (3). Analysis of adaptive correction is conducted below.

Adaptation of the PCP quality functional on the affine family of its Hessians

In this section we consider the following problem. It is necessary to use the regression-tensor model (4) as the basis and construct a numerical procedure of choosing the vector of weighting coefficients $r \in R^n$, which would provide for an elliptic character of the fixed stationary point v^* (a solution of equation (5)) of the goal functional $F(v) = r'w(\omega+v)$, while proceeding from the assumption that algebraic (spectral) conditions (7) are satisfied.

Remark 2: Despite the fact of algebraic equivalence (6) $\sim$ (7), an attempt of usage of expansion of determinants (6) in constructing an adaptive correction $r \to r'w(\omega+v)$ is almost inevitably condemned to failure because there is a large number of terms present in such an expansion.

Necessary and sufficient conditions of solving problem (3) may be obtained only in exceptional cases. As a rule, for such problem statements, the general problem turms out to be *NP*-hard. Below, we intend to discuss an approach to solving this problem for the functional *F*(.). Such an approach is grounded on the provisions of the theory describing localizations and perturbations of eigenvalues of the matrix from [14]. Transformation of conditions (7) to the so called problem of quadratic stability represents another efficient technique. The problem of quadratic stability is usually reduced to constructing the Lyapunov function in the affine family of matrices under the assumption that this family, in turn, is functionally (due to the second formula in (3)) dependent on coordinates of vector $r \in R^n$ [19].

Let there be given an initial vector $r_0 \in R^n$ of weighting coefficients from (3). For example, the process of goal-oriented choosing of r_0 may be realized on account of equality of its coordinates $r_{0i}, 1 \le i \le n$ to the values of some (given) functions $\Psi_i : R \to R$ of functional

$$J_i(v) := w_i(\omega+v), i=1, \ldots, n$$

in the "auxiliary problems" related to forecasting PCP quality with respect to some indicators w_i, $1 \le i \le n$. Due to Corollary 2, for a bivalent model of regression (1) this situation is formalized in the following proposition [5].

Proposition 2: *When k = 2, the initial weighting coefficient vector* $r_0 = col(r_{01}, \ldots, r_{0n})$ *with the coordinates*

$$r_{0i} = \Psi_i(z_i), z_i = max\{J_i(v) : v \in R^m\},\ 1 \le i \le n,$$

has the following analytical representation

$$r_0 = col(\Psi_1(c_1 - e_1' AB_1^{*-1} A' e_1 / 2),\ \ldots, \Psi_n(c_n - e_n' AB_n^{*-1} A' e_n\ / 2)).$$

Remark 3: The statement "*when k* = 2" is not the key one because the given construction of vector r_0 may be used also in the case of the 3-valent (with respect to the predictors) form of regression-tensor model (4); obviously, in this case, r_0 may be "corrected", on account of the condition of normalization $\| r_0 \|_{R^n} = 1$.

Next, denote by $v^0 \in R^m$ a critical point of functional *F*(.) (a fixed solution of equation (5)) in the position, when $r = r_0$; by $G_0 \in M_{m,m}(R)$ denote the Hessian of functional *F*(.) computed at point v^0; now let

$$G_i := B_i^* \ [\partial^2 f_i^{3,m}(v, \ldots, v) / \partial v_g \partial v_p |_{v^0}],\ 1 \le i \le n.$$

Hence, in case of varying of vector *r* according to the condition $r_i = r_{0i} + \Delta r_i > 0,\ 1 \le i \le n$, the parametric family of Hessians *G(r)* from Proposition 1 is defined by the following affine matrix manifold of the form

$$G(r) = (G_0 + \sum_{i=1,\ldots,n} \Delta r_i G_i) \in M_{m,m}(R); \tag{8}$$

For any $r_0 + \Delta r \in R^n$ Hessians (8) are symmetric matrices [14].

In case of an arbitrary matrix, the only description its eigenvalues presumes that these are solutions of its characteristic equation. Obtaining eigenvalues for the Hessian *G(r)* may also be (due to the Courant–Fischer Theorem, characterized as solving an optimization problem [14]. The sphere of possible interpretations of the Courant–Fischer Theorem includes the speculations of Weyl's Theorem on the relations between the eigenvalues of Hessian G_0 and any Hessian from the manifold

$$G_0 + \sum_{1 \le i \le n} \Delta r_i G_i.$$

This allows one to understand the following "variation" sense of the robust-adaptive constructions needed for correction of

$r \to r' w(\omega + v)$ [14]. On account of the constructions introduced above, the potential of robust-adaptive adjustment of functional which provides for satisfaction of inequality (7) at the critical point (in case of varying $r \in R^n$), contains Proposition 3. A modification of Theorem 6.3.12 on account of Theorem 4.1.3, which takes account of symmetric structure of Hessians (8), is implied [14].

Proposition 3: *Let* $\{(\lambda_p(r_0), x_p) : p = 1, \ldots, m\} \subset R \times R^m$ *be eigenpairs of Hessian* G_0 *and*

$g_{pi} = x'_p G_i x_p / x_p' x_p$.

Hence the characteristic numbers $\{\lambda_p(r) : p = 1, \ldots, m\} \subset R$ *of Hessian G(r), where* $r = r_0 + \Delta r$, *write*

$$\lambda_1(r) = \lambda_1(r_0) + \sum_{i=1,\ldots,n} g_{1i}\Delta r_i + o(\| \Delta r \|_{R^n}),$$
$$\lambda_m(r) = \lambda_m(r_0) + \sum_{i=1,\ldots,n} g_{mi}\Delta r_i + o(\| \Delta r \|_{R^n}). \tag{9}$$

System (9) gives the possibility to assess how sensitive the eigennumbers of Hessians (8) to variations of the weighting coefficients $\Delta r_i,\ 1 \le i \le n$, are. Obviously, this analysis is approximate (it is valid only for small $\| \Delta r \|_{R^m}$; see also the formulas of the perturbation theory, what, on account of Corollary 1, is expressed in the following Corollary [16].

Corollary 2: *If k = 2, n = m,* $\Lambda(r_0) := col(\lambda_1(r_0), \ldots, \lambda_m(r_0))$ *is a vector of eigenvalues of Hessian matrix* $(r_{01}B_1^* + \ldots + r_{0m}B_m^*)$ *and* $\{x_p\}_{p=1,\ldots,m}$ *are eigenvectors corresponding to them,* $\Lambda^* := col(\lambda_1^*, \ldots, \lambda_m^*)$ *is the vector of characteristic numbers of Hessian G(r), which are standard with respect to criterion (7),* $B := [b_{pi}]$ *is an m×m-matrix with elements*

$$b_{pi} = x'_p x_p / x'_p x_p,$$

Then for $r = r_0 + \Delta r$ *and* $r_{0i} + \Delta r_i > 0,\ 1 \le i \le m,$ *where* $\Delta r = B^{-1}(\wedge^* - \wedge(r_0)),$ *it is possible to expect that eigenvalues of G(r) are equal to standard ones* $\{\lambda_p^* : p = 1, \ldots, m\}$.

Remark 4: Since system (9) is valid for a small value of $\| \Delta r \|_{R^m}$ the following issue remains open. It is not clear whether the iteration process of the form

$$r_j = (r_{j-1} + \Delta r_{j-1}) \in R^m, j = 1, 2, \ldots ,$$

Constructed due to Corollaries 1, 2 on account that

$$\Delta r_{j-1} = B^{-1}(\wedge^* - \wedge(r_{j-1})),$$

Converges, when the initial divergence $\| \wedge^* - \wedge(r_0) \|_{R^m}$ is substantial or not. Obviously, due to structure of functional (3), it is necessary to verify conditions $r_{ij} > 0,\ 1 \le i \le m$ on each iteration step "*j*" for the coordinate vector $r_j \in R^m$.

Now, while treating the situation in the context of Remark 4, consider the result of computing the upper estimate for the relative perturbation $\| \Delta r \|_{R^m}$. Let $\| . \|_M$ be a matrix norm in $M_{m,m}(R)$ coordinated with $\| \cdot \|_{R^m}$, furthermore, $\|E\|_M = 1$,

where $E \in M_{m,m}(R)$ is a unit matrix; for example, the Frobenius matrix norm is [20].

$$\| D \|_F := (m^{-1} \sum d_{ij}^2)^{1/2}, D = [d_{ij}] \in M_{m,m}(R),$$

And the spectral (induced) matrix norm [20].

$$\| D \|_S := sup\{\| Dx \|_{R^m} : x \in R^m, \| x \|_{R^m} = 1\} = \max_{1 \le i \le m} \lambda_i^{1/2}(D'D).$$

Thus, when turning back to Corollary 2, we have (due to the prototype of system (9)):

$$B\Delta r = \wedge^* - \wedge(r_0)$$

With det B ≠ 0. Suppose, vector $\wedge^* - \wedge(r_0)$ transforms into $\wedge^* - \wedge(r_0) + \delta$ (in particular, at the expense of term $o(\| \Delta r \|_{R^m})$ of system (9)) and matrix *B* transforms into *B* + *D*. In such a problem statement, the vector of adaptive adjustment Δr obtains (due to modification of Corollary 2) increment θ, and assumes the form $\Delta r+\theta$, which satisfies the following linear algebraic equation:

$$(B+D)(\Delta r+\theta) = \wedge^{*}-\wedge(r_0) + \delta.$$

Obviously, vector $\delta \in R^{m}$ and matrix $D \in M_{m,m}(R)$ model perturbations of a "desired change" of the vector of eigen numbers $\wedge^{*}-\wedge(r_0)$, as well as the inaccuracy of parametric assessment of matrix B (note, if $\| D\|_M \| B^{-1}\|_M < 1$, then $\| D\|_M < \| B\|_M$ [20, p. 197]).

Details of the approach to computing the upper estimate of relative perturbation $\|\theta\|_{R^m} / \|\Delta r\|_{R^m}$ are formulated in Corollary 3 (technical details may be found in [20].

Corollary 3: *Let (in addition to assumptions of Corollary 2)*

$$s(B) := \| B\|_M \| B^{-1}\|_M$$

Be a conventional number of matrixes B, where $\|.\|_M$ *is the matrix norm equal to* $\|.\|_F$ *or* $\|.\|_S$*. Hence the following analytical estimate is valid [20]*

$$\|\theta\|_{R^m} / \|\Delta r\|_{R^m} \le s(B)(1 - s(B)\| D\|_M / \| B\|_M)^{-1}(\|\delta\|_{R^m} / \|\wedge^{*}-\wedge(r_0)\|_{R^m} + \| D\|_M / \| B\|_M).$$

If $\|\cdot\|_M = \|\cdot\|_S$ *and* λ_1, λ_m *are, respectively, the smallest eigenvalue B' and the largest eigenvalue B, then it is possible to assume that* $s(B) = (\lambda_m / \lambda_1)^{1/2}$ *in the latter inequality.*

Remark 5: The construction of the spectral conventional number $s(B) = (\lambda_m / \lambda_1)^{1/2}$ (the conventional number obtained with the use of the spectral norm $\|\cdot\|_S$) is transparent due to equality $s(B) = \| B\|_S \| B^{-1}\|_S$.

Conclusion

When the issues of mathematical modeling of complex physics-chemical objects and processes are discussed, the following technological procedure is normally implied: (i) natural differentiation between different aspects of a definitely given PCP (as an object of mathematical investigation), (ii) description of each of the aspects on the basis of one's own (normally, comparatively narrow and easily observable) group of mathematical assumptions, (iii) subsequent integration of the partial results obtained on account of proper specification, (iv) turning back to consideration of the complex (integrated) functioning of PCP.

Within the frames of above paradigm, the idea of the present paper presumes to develop the results of and to point to natural relationship existing between the problem of defining the domain of the matrix Hessian function values at the critical point of its goal functional of physics-mechanical quality (3) for the process of hardening metal coating, expressed by equation (1) and by vector *r* of weighting coefficients in (3), which reflect the "priority" between w_i, $1 \le i \le n$, i.e. between the modeled tribological properties of PCP [5]. In this context, Proposition 1 and Corollary 1 show that, unlike that for the 3-valent (*k* = 3), in the bivalent (*k* = 2) model of nonlinear regression of PCP, the Hessian *G(r)* is invariant with respect to the position of the critical point. In this case, both of the variants (2 = *k* = 3) allow one to reveal the dependence $r \to G(r)$ on the basis of the PCP model (1) identified in course of tribological tests with the aid of criterion (2).

Eigen values of the matrix are definitely the roots of its characteristic polynomial, so, the result of Proposition 3 is, in essence, based on the assumption that eigen values (7) are continuously *r*-dependent on the elements of Hessian matrix *G(r)* in the process of the ongoing parametric correction of the goal functional *F* from (3). Noteworthy, some part of information turns out to be lost, when we deal only with the characteristic polynomial, because there are many different matrices with the given characteristic polynomial. So, no wonder that stronger results obtained in modeling the Hessian's *G(r)* spectrum, in particular, Proposition 3 and Corollary 2, take account of the structure of matrix *G(r)*; the latter assume some technical simplification, which follows from the assumption that any Hessian matrix is orthogonally similar to the real diagonal matrix [20].

Numerical methods of finding eigen values and eigen vectors represent one of the most important divisions of the general matrix theory. The present paper, considers analysis of vector $\wedge^{*}-\wedge(r_0)$ and matrix *B* on account of Corollary 2, and does not touch any aspects of this complicated issue. Meanwhile, Corollary 3 suggests an upper estimate of the relative perturbation *Δr* obtained via the estimate of relative perturbations $\wedge^{*}-\wedge(r_0)$, the estimate of *B* and the estimate of the conventional number *s(B)*. Note, *s(B)* participates in assessment in all the cases, independently whether the perturbations take place only in $\wedge^{*}-\wedge(r_0)$, only in *B* or in $\wedge^{*}-\wedge(r_0)$ and in *B* simultaneously.

In conclusion, let us discuss another approach related to adaptive correction of $r \to r'w(\omega+v)$, which is bound up with the usage of sufficient conditions of *robust stability* of matrix G(r) (what is also equivalent to conditions (6), (7)). In this context, a requirement may be put forward that – in the family $G_0 + \sum_{1\le i\le n} \Delta r_i G_j$ and under the interval constrains imposed on variations of the coordinate vector Δr – it would be possible to construct a Lyapunov function $V(x) = x_p{}' P x_p$, where $P \in M_{m,m}(R)$ is a symmetric positive definite matrix; i.e. an assumption that there would exist some matrix P > 0, for which the Lyapunov matrix equation $G(r)P + PG(r) = -Q$ has a solution for a given symmetric positive definite matrix $Q \in M_{m,m}(R)$. An approach bound up with the transition to adaptive-robust quadratic stability and the methods of solving the problem with such a statement can be found in [21–24]. Owing to the abundance of computational problems described by the proposed theory and due to the great possibilities it opens for solving of multidimensional regression-tensor analysis problems, which presume various applications, this theory may now acquire an important independent value from the viewpoint of applications to the problem of synthesis of optimal coatings. It is hardly ever possible to consider all the aspects in one paper. But we are sure, that further detailed investigations in this direction will follow soon.

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