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On a Loss Distribution Approach for Allocating Operational Risk Capital in Banks

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Abstract

As per the Basel accords (I-III), banks are required to keep a certain amount of capital in reserve to cater for losses resulting from their operational loss events. In estimating capital, banks may use any of the three approaches outlined in the Basel II accord; Basic Indicator Approach (BIS), The Standardized Approach (TSA) and the Advanced Measurement Approach (AMA). This paper serves to suggest robust operational risk capital allocation techniques under the AMA. The quantitative assessment of operational risk here-in is informed by loss distributions of a light-heavy tail nature as well under the Peak-Over-Threshold method. Some graphical and empirical goodness-of-fit techniques are used in selecting the best distribution for each considered operational loss data set. Simulation studies from all considered distributions are carried out to establish as to which goodness-of-fit techniques best fit for what loss distribution. This is an analytical work that attempts to fit several theoretical distributions to real banking and financial data and measure the fitting performance by different approaches. Two sets of data are considered, one from a Spanish Saving bank and another from a South African retail bank. Operational risk capital is calculated based on the allocation of the best-fit distribution for each dataset. The capital charge here-in is estimated as the $100(1-\alpha)^{th}$ percentile (Empirical Value-at-Risk) at level α of the best fit loss distribution.

Keywords: Operational Risk Capital; Advanced Measurement Approach; Loss Distribution Approach; Goodness-of-Fit; Peaks-Over-Threshold; SME Bank Namibia

Introduction

Operational risk is defined as the potential of loss resulting from inadequate or failed internal processes, people and systems or from external events [1]. Over the years major operational losses experienced by financial institutions globally has exceeded \$100 million. The \$691 million rogue trading loss at All first Financial, the \$484 million settlement due to misleading sales prices at household finance and the estimated \$140 million loss from the 9/11 attack at the Bank of New York just to mention but a few are some of the classical global examples.

Locally, there are two prominent banks which draw attention. The City Savings & Investment Bank which collapsed over a decade ago as well as the recently liquidated Small and Medium Enterprises (SME) Bank Namibia. In case of the SME Bank, its operational failure emanates from nonperforming loans, ghost workers, issues of noncompliance, misappropriation of funds, falsified documentations; deceiving and misrepresentation of information to the regulator among other irregularities [2]. To safeguard the stability of banks, the Basel Committee on Banking Supervision (BCBS) outlined three methods for reserving risk capital. The three are; the Basic Indicator Approach (BIS), the Standardized Approach (TSA), and the Advanced Measurement Approaches (AMA).

For synthesis, we briefly discuss these three approaches. Under the BIS approach, operational risk (K_{BIA}) is calculated as the simple average over the previous three years of a fixed percentage (α) of positive annual gross income (GI), where α is fixed at 15% which according to [1] relates to an industry wide level of required capital.

Under the TSA, banking activities are sub-divided into eight business lines: corporate finance, trading & sales, retail banking, commercial banking, payment & settlement, agency services, asset management, and retail brokerage. Within each business line, GI is used as a proxy for operational risk exposure. The capital charge for each business line is calculated by multiplying GI by a factor βi assigned to each business line.

However, under the AMA capital is estimated from the bank's actual internal operational loss data using its own internal rating model. A bank intending to use the AMA should therefore demonstrate the accuracy of its' internal model(s) in assessing risk within the different business lines within that specific bank. Furthermore, the bank should also meet the following criterions; use its own internal loss data, or supplemented by relevant external data rescaled to reflect its actual exposure, use scenarios/stress testing's techniques and any other factors reflecting the actual operations. Moreover, the measures of risk relevant for estimating risk capital should correspond to the 99.9% confidence level of exposure for a one-year holding period. The intention of AMA is to provide incentives to banks to invest into development of sound operational risk practices and risk management methodologies. According to [3] the capital reserve under AMA (when compared to other approaches) is more relevant in reflecting the actual risk profile of the bank. Under the loss distribution approach which falls under the AMA, banks estimate for each business line/risk type combination, the probability distribution functions of the single event impact and the event frequency for the next (one) year holding period using its own internal data [4].

As a requirement per the Basel accords, capital charge is defined at the 99.9% percentile of the total loss distribution less the expected losses or 99.9% percentile of the total loss distribution where only above the threshold losses are considered. The loss distribution approach relies on historical loss data in order to accurately estimate a reliable capital charge. Since not all operational losses are publicly reported by banks, it is very difficult to effectively model bank operational loss data. Also, a majority of existing operational risk databases are heavy populated with high frequency, low severity losses and only a very few high severity losses are featured in those databases. These among other factors greatly contribute to the use of top-down approaches in estimating risk capital. According to [1,3] and references there-in, the use of top-down approaches leads under-estimation of capital.

This paper therefore, serves to suggest a robust methodology for estimating operational risk capital charge using the loss distribution approach. The rest of the paper is structured as follow; in section 4 we discuss different goodness-of-fit techniques necessary for choosing the best loss distribution for the two considered datasets. The goodness-of-fit techniques employed here falls under the two main categories: the graphical and empirical goodness- of-fit techniques. Section 4.3 presents a review of properties of different loss distributions well suited for modelling loss frequency (arrival processes) as well as those appropriate for modelling the severities of bank operational loss data. The section further highlights on procedures for simulating random variates from the considered loss severity distributions. The actual application of the approach, simulation studies as well as fitting of real operational loss data and results therein are presented in Section 5. All considered distributions were fitted with parameters estimated using the maximum likelihood approach. The summary of results as well as conclusions drawn from the study is presented in section 6. To the best of our ability there is no literature suggesting as to which goodness-of-fit techniques best fits for what loss distribution and what are the fundamental graphical features of loss data following a loss distribution from the light-heavy tailed distribution class. We therefore believe this paper is very fundamental in laying a good foundation for further research on operational risk modelling techniques [5-10].

Materials and Methods

To begin, we briefly discuss contemporary definitions and properties of different techniques followed by different loss distributions discussed in the paper.

Some Basic Definitions

A very popular measure of risk is the well-known Value-at-Risk (VaR). VaR is a statistical measure of the riskiness of a given portfolio. It is defined as the maximum dollar amount ex- pected to be lost over a given time horizon, at a pre-defined confidence level. Mathematically, Holton & Glyn [11] defines VaR as follow.

Definition 1: Value-at-Risk

Given a loss random variable L and a confidence level $p \in (0,1)$, $VaR_P(L)$ is given by the smallest number *l* such that, the probability that the loss *L* exceeds *l* is no larger than 1-p, i.e.

$$\begin{aligned} VaR_P(L) &= \inf\{l \in \mathbb{R} : P(L > l) \le 1 - p\} \\ &= inf\,\{l \in \mathbb{R} : 1 - F_L(l) \le 1 - p\} \\ &= inf\,\{l \in \mathbb{R} : F_L(l) \le p\}, \end{aligned} \tag{1}$$

where $F_L(l)$ is the probability distribution function for the random variable L.

Graphical Goodness-of-Fit Technique

Definition 2: Quantile-Quantile Plot (QQ-plot)

The QQ-plot, plot empirical quantiles against the quintiles of a hypothesized distribution for the data.

If the distribution is chosen correctly, the QQ plot should coincide with the line y = x in the xy-plane. Otherwise, if the data follow a heavy tailed distribution the QQ plot would capture it by curving below or above the line y = x.

Definition 3: Mean Excess Plot

For a specified high threshold value *u*, the mean excess function of *x* is given by

$$e(u) = \mathrm{E}[x-u \mid x>u], \tag{2}$$

The sample mean excess function is calculated as

$$\hat{e}_n(u) = \frac{\sum_{j=1}^{n}\left(x_j-u\right)}{\sum_{j=1}^{n} \| \{x_j>u\}}, \tag{3}$$

In simple terms, the mean excess function calculate the expected value of losses above a given high threshold *u*. For heavy-tailed data *e(u)* typically tends to infinity with an upward sloping mean excess plot.

Empirical Distribution Function based Goodness-of-Fit Tests

Empirical distribution function based tests directly compare the empirical distribution function with the fitted distribution function. We start with the popular Kolmogorov-Smirnov test.

Definition 4: Kolmogorov-Smirnov (K-S) Test

This test is used to decide if a sample comes from a population with a specific distribution.

The K-S test is based on the empirical distribution function (ECDF)

$$E_n = n(i)/N,$$

Given (Y_1, Y_2, ..., Y_n) is an ordered set of N data points, and n(i) is the number of points less than Y_i with (Y_1, Y_2, ..., Y_n) ordered from smallest to largest. The test statistic is calculated as follow

$$D = \max_{1\le i\le N}\left(F(Y_i) - \frac{i-1}{N}, \frac{i}{N} - F(Y_i) \right), \tag{4}$$

where F is the theoretical cumulative distribution of the continuous distribution being tested.

The null hypothesis (that, the data follows the hypothesized distribution with a cumulative distribution function F) is rejected if the test statistic D is greater than the critical value.

Definition 5: Anderson-Darling Test

The Anderson-Darling test is a modification of the K-S test and gives more weight to the tails of the distribution of the data compared to the K-S test [12]. There are two types of Anderson-Darling tests: the supremum type and the quadratic type. The supremum type AD statistic with empirical distribution function (EDF) F_n*(x)* and fitted distribution function *F(x)* is calculated as

$$AD = \sqrt{n}\, \sup_x \left| \frac{F_n(x)-F(x)}{\sqrt{F(x)(1-F(x))}} \right|, \tag{5}$$

with computing formula

$$AD = \sqrt{n} \max\left\{ \sup_j \left\{ \frac{\frac{j}{n} - z_{(j)}}{\sqrt{z_{(j)}(1-z_{(j)})}} \right\}, \sup_j \left\{ \frac{z_{(j)} - \frac{j-1}{n}}{\sqrt{z_{(j)}(1-z_{(j)})}} \right\} \right\} \tag{6}$$

The quadratic type AD statistic is calculated as

$$AD^2 = n\int_{-\infty}^{+\infty} \frac{(F_n(x) - F(x))^2}{F(x)(1-F(x))} dF(x), \tag{7}$$

with computing formula

$$AD^2 = -n + \frac{1}{n}\sum_{j=1}^{n}(1-2j)log z_{(j)} - \frac{1}{n}\sum_{j=1}^{n}(1+2(n-j))log(1-z_{(j)}) \tag{8}$$

Since this test puts more weights on tails of distributions, it is important when there is reason to believe that the underlying data is heavy-tailed.

Definition 6: Cramer-von Misses Test (W^2)

The Cramer-von Misses test statistic [13] is defined by

$$W^2 = n\int_{-\infty}^{+\infty} (F_n(x) - F(X))^2 dF(X),$$

with computing formula

$$W^2 = \frac{n}{3} + \frac{1}{n}\sum_{j=1}^{n}(1-2j)z_{(j)} + \sum_{j=1}^{n} z_{(j)}^2$$

Definition 7: Kuiper Test (V_n)

Kuiper proposed V_n, an adaptation of the Kolmogorov statistic to test the null hypothesis that a random sample of size N comes from a population with given continuous distribution function $F(x)$. If the sample distribution function is $F_N(x)$, V_N is defined by

$$V_n = \sup_{-\infty<x<\infty}(F_N(x) - F(x)) - \inf_{-\infty<x<\infty}(F_N(x) - F(x)).$$

Loss Distributions

Under the loss distribution approach (LDA) banks are required to quantify two sets of distributions, the loss frequency and loss severity distributions for each risk cell (business line/event type combination) over a one-year time horizon. Though the loss frequency distributions are important for modelling the arrivals and frequencies of losses, this paper focus on modelling the severity of the loss data.

Loss Severity Distributions and their Simulations

This section therefore present the different methods involved in building robust data driven loss severity models.

Quantiles Estimation

One common method for simulating from loss distributions or estimating high quantiles is the inverse transform method.

Definition 8: Inverse Transform Method

Suppose $F(x):\mathbb{R}\to[0,1]$ is a non-negative and non-decreasing cumulative loss distribution function for a given loss data. The inverse transform method involves find a quantile expression for the distribution function F, which is obtained from the generalized inverse $F^{-1}(y):[0,1]\to\mathbb{R}$ of F given by

$$F^{-1}(y) = min\{x : F(x) \geq y\}, y \in [0,1] \tag{9}$$

$F^{-1}(y)$ gives the estimated loss value occuring with a probability $y \in [0,1]$. It is however worth mentioning that, one can only use the method on condition that $F^{-1}(y)$ can be explicitly expressed in a closed form.

Lognormal Distribution

The lognormal distribution is very useful in modelling of claim/loss sizes. It has thick tails, is right skewed and and such fits many phenomenon's with positive supports. It resembles the normal distribution when the standard deviation (σ) is small; it is infinitely divisible and closed under power and scale transformations.

For a random variable x with normal distribution

$$f_N(x) = \frac{1}{\sqrt{2\pi\sigma}} \exp\left\{-\frac{1}{2}\frac{(x-\mu)^2}{\sigma^2}\right\}, -\infty < x < \infty \tag{10}$$

Let $y = e^x$ such that $X = logy$, then the probability density function of y is given by:

$$f(y) = f_N(logy)\frac{1}{y} = \frac{1}{\sqrt{2\pi\sigma}} \exp\left\{-\frac{1}{2}\frac{(logy-\mu)^2}{\sigma^2}\right\}, y > 0 \tag{11}$$

where $\sigma > 0$ is the scale parameter and $-\infty < \mu < \infty$ is the location parameter. The distribution of y is called lognormal or Cobb-Douglass law [14]. The lognormal cdf is given by

$$F(y) = \Phi\left(\frac{logy-\mu}{\sigma}\right), \mathrm{y} > 0 \tag{12}$$

with the mean and variance given by

$$E(y) = \exp\left(\mu + \frac{\sigma^2}{2}\right), \tag{13}$$

$$Var(y) = \left[\exp\left(\sigma^2\right) - 1\right]\exp\left(2\mu + \sigma^2\right) \tag{14}$$

respectively, with the right tail behavior of the lognormal given by

$$\bar{F}(y) = 1 - \Phi\left(\frac{logx}{\quad}\right). \tag{15}$$

The inverse transform of the lognormal distribution is given by

$$F^{-1}(y) = e^{\Phi^{-1}(u)\sigma+\mu} \tag{16}$$

for $p \in (0,1)$. Therefore, the lognormal distribution with scale parameter μ and location parameter σ has the following quantile function

$$\mathrm{Q(p)} = e^{\Phi^{-1}(\mathrm{p})\sigma+\mu} \tag{17}$$

The lognormal QQ-plot should show a straight line with slope $\hat{\sigma}$ and intercept $\hat{\mu}$. If large magnitude losses are plotting below the fitted line, then it is an indication that the data is not well represented by a lognormal distribution, hence a more heavy tailed distribution should be considered. For the mean excess function, the lognormal distributed data should yield a mean excess plot with an upward sloping curve.

Weibull Distribution

The Weibull is a generalization of the exponential with two parameters instead of one allowing for greater flexibility and heavier tails. The density and distribution functions are

$$f(x) = \alpha\beta x^{\alpha-1}e^{-\beta x\alpha}, x \geq 0 \tag{18}$$

$$F(x) = 1 - e^{-\beta x\alpha}, x \geq 0 \tag{19}$$

where $\beta,\alpha \geq 0$ are the scale and shape parameters respectively. The mean and variance of the Weibull are

$$E(x) = \beta^{-\frac{1}{\alpha}}\Gamma\left(1+\frac{1}{\alpha}\right), Var(x) = \beta^{-\frac{2}{\alpha}}\left(\Gamma\left(1+\frac{2}{\alpha}\right)-\Gamma^2\left(1+\frac{1}{\alpha}\right)\right) \tag{20}$$

It's survival function is given by $\bar{F}(x)=1-F(x)=e^{\beta x^{\alpha}}$ which makes it heavy-tailed when α≤1.

To generate a Weibull random variate, one first generates an exponential random variable Y with parameter β and then follows the transformation $X = Y^{1/\alpha}(\alpha,\beta)$ to obtain Weibull (α,β) random variate.

The Weibull quantile function with scale parameter α and shape parameter β is given by

$$Q(p) = \beta\left(-log(1-p)^{1/\alpha}\right) p \in (0,1) \tag{21}$$

A Weibull QQ-plot has a slope approximately equal to $1/\hat{\alpha}$ and intercept $\log(\hat{\beta})$. If large magnitude losses plot below the fitted line, then Weibull does not make provision for suchlosses and a distribution with heavier tails than the Weibull should be considered. On the other hand, a Weibull mean excess plot should give a downward sloping curve.

Pareto Distribution

Suppose a random variable X, has an exponential distribution with mean λ^{-1}, and that λ itself has a gamma distribution with parameter α, then the unconditional mixture distribution of x is called the Pareto distribution. The density and distribution functions of the Pareto are given by

$$f(x) = \frac{\alpha\lambda^{\alpha}}{(\lambda+x)^{\alpha+1}}, x>0 \tag{22}$$

$$F(x) = 1-\left(\frac{\lambda}{\lambda+x}\right)^{\alpha}, x>0 \tag{23}$$

respectively, where α is the shape parameter λ and the scale parameter. The mean and variance are given as

$$E(x) = \frac{\lambda}{\alpha-1} \tag{24}$$

and

$$Var(x) = \frac{\alpha\lambda^2}{(\alpha-1)^2(\alpha-2)} \tag{25}$$

respectively, where the mean only exists for $\alpha>1$ and the variance only exists for $\alpha>2$. The inverse of the cdf has a simple analytical form

$$Q(p) = \lambda\left\{(1-p)^{-1/\alpha}-1\right\}, p \in (0,1) \tag{26}$$

If the data follows a Pareto distribution we expect the Pareto QQ-plot to be linear but only in some of the largest observations. The mean excess plot for the Pareto should also give an upward sloping curve.

Generalized Pareto Distribution (GPD)

The generalized Pareto distribution can be used for modelling tails events, i.e for data exceeding a certain high threshold. For a loss random variable x , if we choose a threshold u, and denote $x-u$ by exceedances of x over a given threshold u, then the excess distribution function is given by

$$F_u(x) = P\left(x - u \le x \middle| x > u\right) = \frac{F(x+u) - F(u)}{1 - F(u)} \tag{27}$$

and the generalized Pareto distribution (GPD) which is the best candidate for modelling such exceedances has distribution function

$$G_{\xi,\beta} = \begin{cases} 1-\left(1+\xi\dfrac{x-\mu}{\beta}\right)^{-1/\xi} & if\,\xi\neq 0, \\ 1-e^{\frac{x-\mu}{\beta}} & if\,\xi=1 \end{cases} \tag{28}$$

whereby $\beta > 0$ is the scale parameter, μ the location parameter and ξ the shape parameter see [4] and reference therein.

The inverse of the GPD takes the form $F^{-1}(p) = \mu - \beta log(1-p)$ for $\xi = 0$ and $F^{-1}(p) = \mu - \beta log(1 - p^{-\xi})/\xi$ for $\xi \neq 0, p \in (0,1)$ The quantile expressions for the GPD are given by

$$Q(p) = \mu - \beta log(1-U) \text{ for } \xi = 0 \tag{29}$$

and

$$\text{Q(p)} = \mu - \beta log(1 - U^{-\xi})/\xi \text{ for } \xi \neq 0 \tag{30}$$

whereby U is uniformly distributed on (0,1).
The mean excess function for the GPD is given by

$$e(u) = \frac{\beta}{1-\xi} + \frac{\xi}{1-\xi}u, \tag{31}$$

which means that for 0<ξ<1 and β+ξu>0 the mean excess plot should have an upward-sloping straight curve.

Note that, the mean excess plot is more steeper for heavy tailed data and an upward sloping plot would indicate a Pareto-like distribution, while a horizontal plot would indicate an exponential distribution.

The optimal threshold is chosen such that, the mean excess plot is roughly linear for $x \ge u$. If one uses the empirical distribution function for losses below the threshold, then $(n - N_u)/n$ losses should fall below the threshold u and N_u losses above it, where n is the number of losses observed with a one year period.

Therefore, the Value-at-Risk estimate under the GPD method, tis given by

$$Va\hat{R}_{1-\alpha} = u - \frac{\hat{\beta}}{\hat{\xi}}\left(1 - \left(\frac{N_u}{n(1-\alpha)}\right)^{\hat{\xi}}\right). \tag{32}$$

This approach has come to be known as a Peaks-Over-Thresholds (POT) method.

Results and Discussion

Before fitting the distributions to the two considered sets of operational loss data, we carried out simulation studies on all considered loss distributions. Thereafter we tested all simulations for goodness-of-fit using different goodness-of-fit techniques discussed in the previous section. The simulations were carried out using the inverse transformation of the respective distribution functions. After establishing which goodness-of-fit test best fit for what distribution, we fitted all considered distributions to the actual observed operational loss data. The parameters in the fitted distributions were estimated using the maximum likelihood method as it provided consistent unbiased estimates as compared to the least square method. The data under consideration consists of two datasets. One provided by a medium-sized Spanish savings bank and another one from a South African retail bank. The choice of the data was solemnly based on the availability. All datasets consist of daily observations captured with reference to the date of occurrence. However, this study used monthly aggregated observations. The Spanish losses were captured in Euros (€) while South African ones in Rands (R). In the subsequent subsections we will first discuss the Spanish data and then the South African one.

Spanish Savings Bank Data

The Spanish data represent losses incurred by a medium sized Spanish Savings Bank and were obtained from [15].

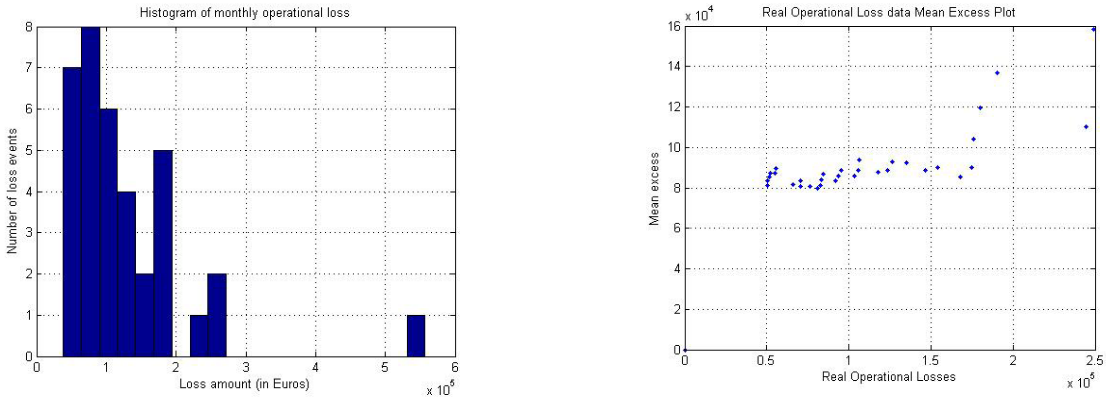

Figure 1: Spanish savings bank data histogram and mean excess plot

From the histogram in Figure 1, we see that, most losses are of low magnitude, ranging between €50,000 and €200,000. While an extreme loss event of magnitude over €550,000 can be observed far in the right-tail of the histogram, operational losses such as this one are often of an unexpected nature and as such they may cause modelling challenges, since often they tend to be differently distributed than the rest of the dataset. The mean excess plot form a constant function for losses between €50,000 and €150,000, feature of exponential data. However, beyond €150,000 the excess function is increasing, and this is a typical feature of heavy tailed data. It is therefore evident that, the tail distribution of this dataset is different from the distribution in the body. Hence the need to consider light-to-heavy tailed distributions in quantifying high quantiles of the data.

Lognormal Fit

The lognormal fits fairly well to the data with the best goodness-of-fit provided by the Cramer-von Misses test with a test statistic of 0.032267 at 5% significance level. The first figure in Figure 2 indicates that there is a fairly good fit between the empirical cdf and the fitted lognormal cdf. The QQ-plot on the right hand side of Figure 2 corresponds well with a typical lognormal QQ-plot as discussed in the previous section (see section 4.3.2). Results here therefore suggest that, indeed the lognormal distribution does provide a fairly good fit to the Spanish data.

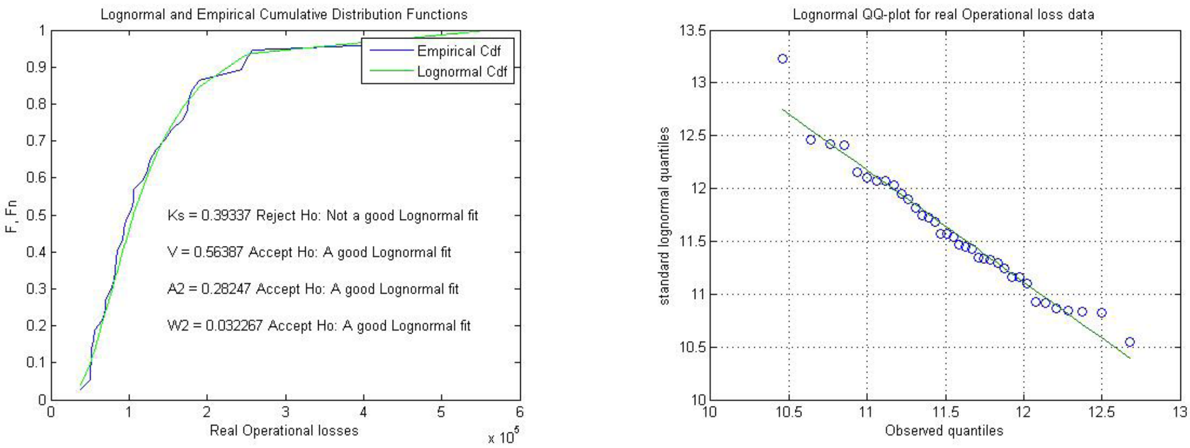

Figure 2: Lognormal fit and QQ-plot to Spanish savings bank data

Weibull Fit

Observing the Figure 3 below, we see that, the Cramer-von Misses test with a test statistic of 0.152 indicates a good Weibull fit for the data. Furthermore, the QQ-plot herein also seems to suggest that the Weibull distribution underestimates some of the high quantiles. These are an indication that, the Weibull distribution is not the best distribution to consider as we look to avoid underestimating capital.

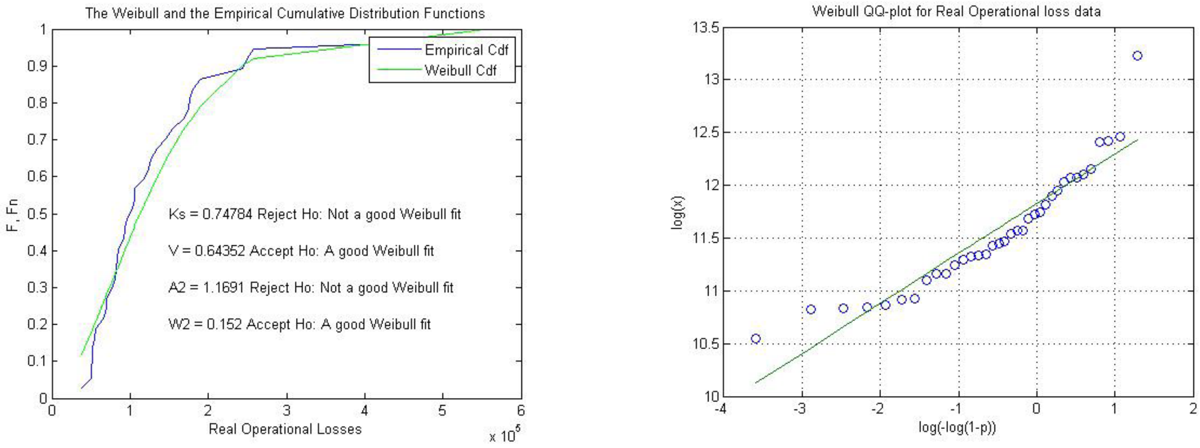

Figure 3: Weibull fit and QQ-plot to Spanish savings bank data

Pareto Fit

We established from the random variates simulation that the Kuiper V test gives the best Pareto fit with the lowest test statistic. We see here that the Kuiper test accepts H_0 giving a good Pareto fit but we observe from the Figure 4 that though that is the case, it is not an optimally good fit. This can be because of the size of our data set and the Pareto being heavy-tailed works fairly well with data exceeding high thresholds. We observe from the QQ-plot that the Pareto is overestimating lower quantiles and underestimating the higher quantiles of the data.

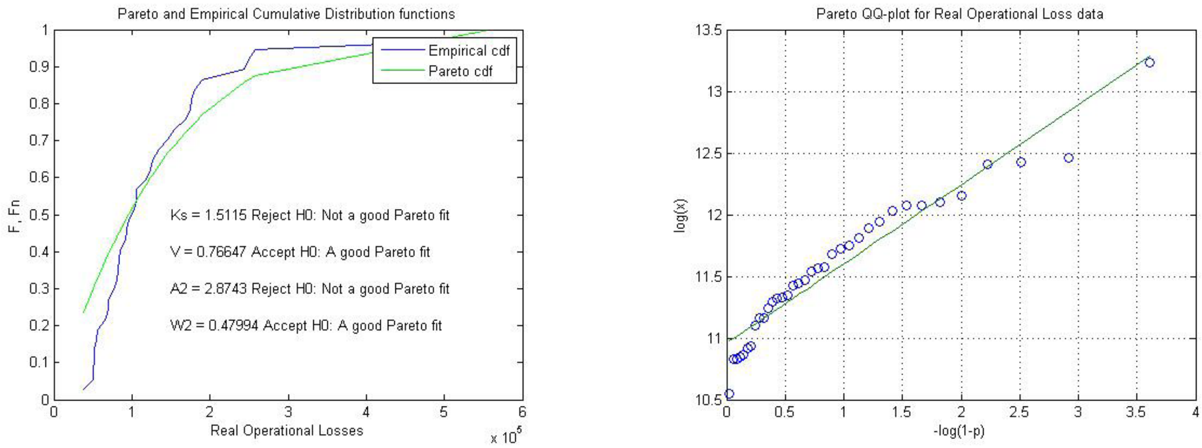

Figure 4: Pareto fit and QQ-plot to Spanish savings bank data

Value-at-Risk

We now look at the Value-at-Risk (VaR) given by the three distributions fitted above. The graph below shows all three distributions and how they compare with each other based on the VaR they estimate.

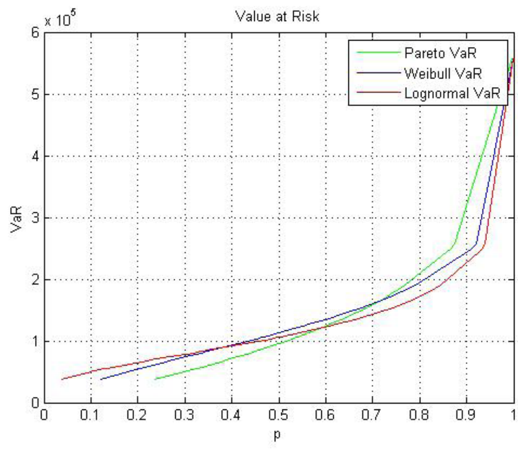

Figure 5: Value-at-Risk for Spanish Savings Bank data

We have on the Y axis the estimated Value-at-Risk and on the X axis the confidence interval. We observed form fitting the data to the distributions that, the lognormal fits better to our data as compared to the other two distributions. We also observe from Figure 5 that, the lognormal and the Pareto distributions do not give a 99.9% quantile estimation for the Value-at-Risk, but we do observe an estimation of the Value-at-Risk given at €557100.00 by the lognormal and Pareto distributions at 99.8% and 99.7% confidence levels respectively. This means that, if we consider the lognormal estimation, we are 99.8% confident that at any given point in time, our losses would not exceed €557100.00, and 99.7% confident it would not exceed the same amount under the Pareto distribution. However, we obtained a 99.9% VaR estimation of €557100.00 using the Weibull distribution. Having estimated the 99.9% quantile with the Weibull and having reached the same estimation with the lognormal which fits best to our data sooner, we can conclude that €557100 is the amount of capital the Spanish Savings bank has to put aside to cater for operational losses. We do however consider the fact that, these distributions fit well to the body of the data but not soo well to the tail, as such, extreme losses are not well captured using the above distributions.

South African Retail Bank Data

The second dataset from a South African retail bank was obtained from [16]. Similarly we briefly study the basic properties of the data.

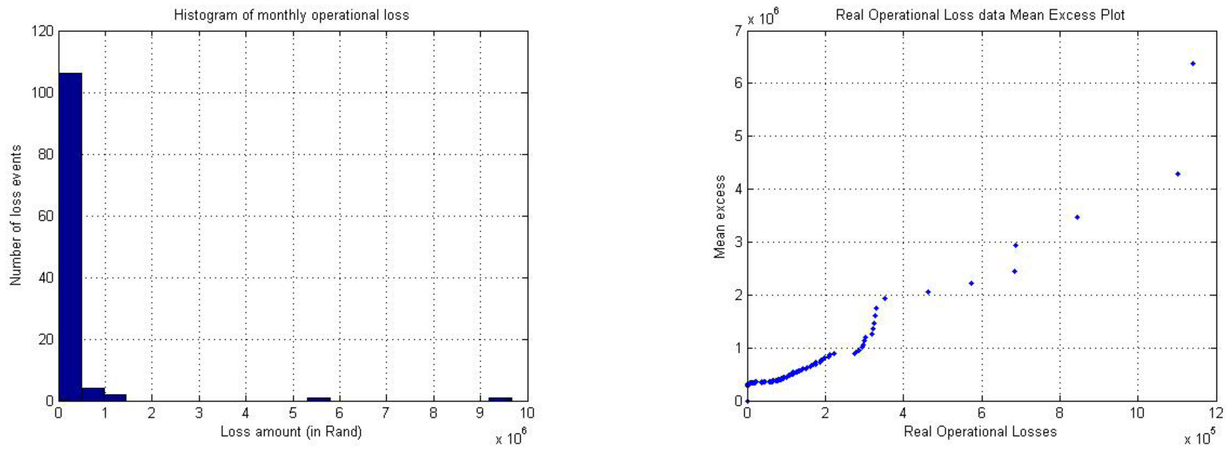

Figure 6: Histogram and mean excess plot of SA retail bank

From the histogram in Figure 6, similar to the Spanish data, and as in literature [15] we observe that, a majority of losses have low severity, with a huge volume of the losses of magnitude less than R500,000.00. We also observe some heavy-tail behaviours, with some extreme losses ranging between R5 million and R6 million and a very extreme observation with magnitude over R9 million. The mean excess plot of the data shows a somewhat slower increasing function in the low magnitude area which increases more rapidly past the R 270,000.00 threshold depicting that our data follows a heavy-tailed distribution.

Lognormal Fit

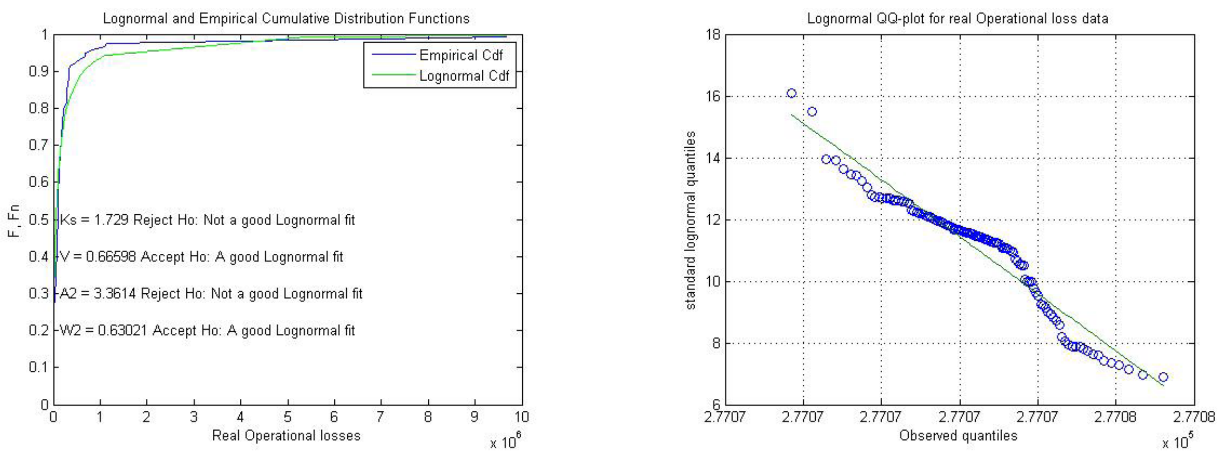

Figure 7: Lognormal fit to SA retail bank data

As in the case of the Spanish savings bank's data considered earlier, the lognormal provides a very good fit to the data, though not as a good-fit as with the Spanish bank's data. The Cramer-von Misses test accepts the lognormal distribution with the smallest test statistic of W2 = 0.63021. However the AD and KS tests rejected the lognormal fit. Graphically, the QQ-plot on the right of Figure 7 shows a reasonably good lognormal fit to the data, though there are still some data points not corresponding with the fitted line. Therefore, a more heavier-tailed distribution may be an appropriate candidate for this dataset.

Weibull Fit

Compared to the Lognormal distribution in Figure 7, Figure 8 indicates that, the Weibull distribution fits the data better, with the best best statistic provided by the Kuiper-statistic V. This is well in accordance with results obtain from the simulation study in accepts our Weibull fit with a test statistic of V = 0.61875, but was however not the smallest statistic with the Cramer-von Misses test giving the best test statistic at W2 = 0.37674. We see further from the QQ-plot on the right that for higher quantiles we tend to have a good fit for the Weibull but we do have some data points plotting bellow the fitted line. The Weibull fits well but not for all the data as we still have a significant amount of data points deviating from the fitted Weibull distribution [17].

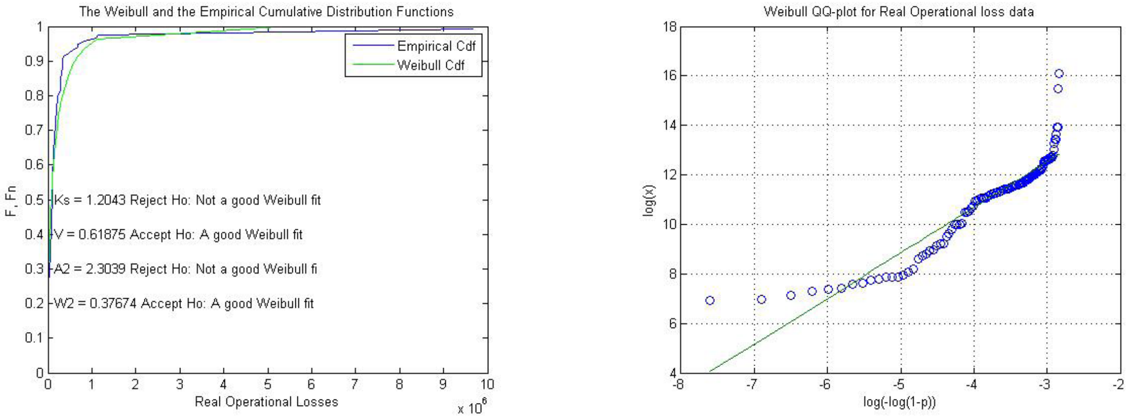

Figure 8: Weibull fit to SA retail bank data

Pareto Fit

For the Pareto distribution in Figure 9, the Kuiper V test gives a good Pareto fit though the Cramer-von Misses test overall is the best with a smallest value of W2 = 0.33482. We further observe from the QQ-plot a considerable number of the extreme values plotting away from the fitted line, hence suggesting for a distribution much heavier than the fitted Pareto distribution.

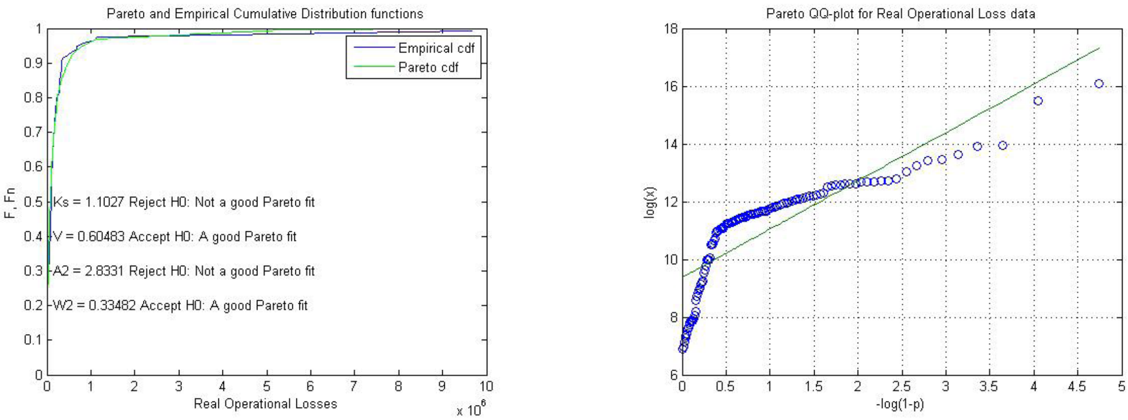

Figure 9: Pareto fit to SA retail bank data

Value-at-Risk

We have on the graph below the Value-at-Risk estimations for the South African Retail Bank's data for each of the fitted loss distributions

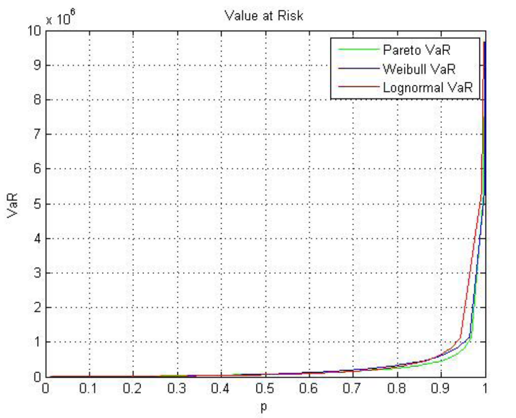

Figure 10: Value-at-Risk for South African Retail Bank data

The Value-at-Risk estimates in Figure 10 shows that the Pareto distribution provides the best estimates compared to the other two loss distributions. We observe the 99.8%VaR from the Pareto estimated at R 9,669,000.00, meaning we are 99.8% confident that no loss will exceed this amount. Whereas, the Weibull give a 99.9% quantile estimate of the Value- at-Risk at R 5,380,000.00. The lognormal estimates the 99.7% Value-at-Risk at R 9,669,000.00. We also observe here that, the Weibull estimate is much lower than those of the other two considered loss distributions. These implies that the Weibull distribution does not well capture extreme losses. Further investigation is needed for us to obtain a much better estimate of the VaR for this data set using the POT method. Should we consider the Pareto distribution which fitted best to the data, we would expect the SA Retail Bank to set aside R 9,669,000.00 as their operational risk capital charge [18].

Peaks-Over-Threshold

The peaks over threshold (POT) method is used in modelling losses beyond high thresholds, in order to capture the effects of extreme losses/event on the estimation of VaR and many other risk measures. From the mean excess plots in Figures 1 & 4 we observe that, due to the presence of extreme losses, the tails and the body of the two datasets are independently distributed. As such, in the following subsection we analyze our datasets using the POT method.

The optimal high thresholds u as previously discussed in section 4.3.5 is chosen at a point where the mean excess plot is roughly linear for $x \geq u$.

Peaks over Threshold for Spanish Savings Bank's Data

Looking at the mean excess plot of the Spanish savings bank's data in Figure 1, we choose a threshold u= €150000, since we see that the distribution of our data past that threshold tends to be different from data bellow that point. We fit the data exceeding this point to the Generalized Pareto distribution (GPD) and analyse the fit (Figure 11).

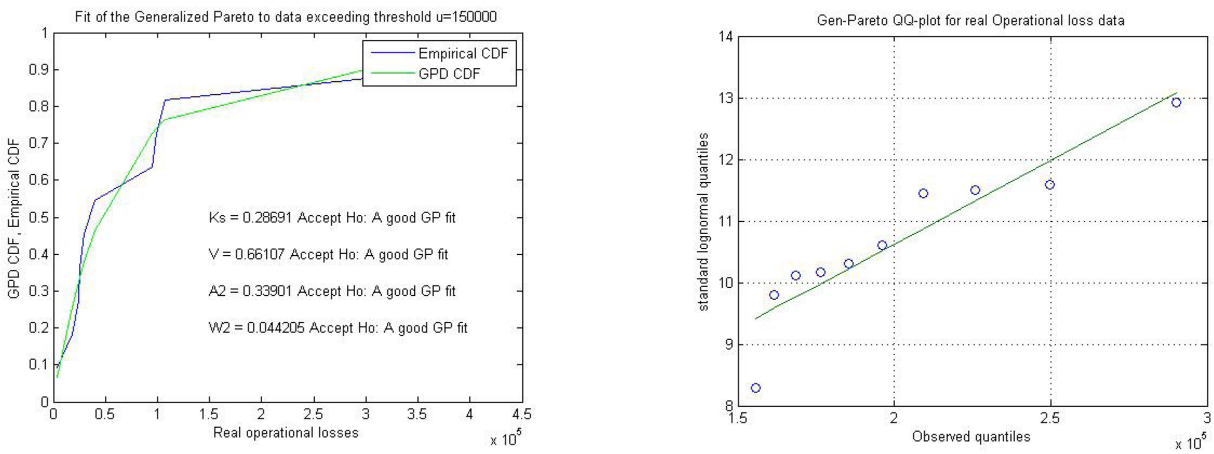

Figure 11: Peaks over threshold of Spanish savings bank's data

We observe from the empirical fit that the data fits well to the GPD with all the considered goodness-of-fit tests accepting fit. The Cramer-von Misses test gave the smallest and best fit with a test statistic of W2 = 044205. On the right-hand side, the QQ-plot indicates that, the GPD does provide a good fit to the data with all data points plotting along the fitted line.

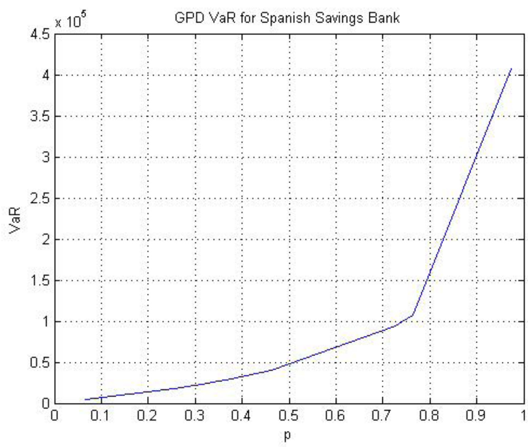

Figure 12: POT Value-at-Risk for Spanish Savings Bank data

Figure 12 indicates that at 97.4% confidence level, we are confident that no extreme losses beyond €150,000.00 incurred by the Spanish bank will exceed €407,100.00 had been breached. Thus due to the POT estimate the bank should set aside this amount to cater for extreme losses. This high severity low frequency loss capital charge along with the low severity high frequency capital charge estimated earlier, the bank should be in a good position to cover itself from operational loss on all fronts [19].

Peaks over Threshold of SA Retail Bank's Data

We observe the mean excess plot of the SA retail bank's data in Figure 5 and choose a threshold u =R280,000 and fit the exceedances to the GPD (Figure 13).

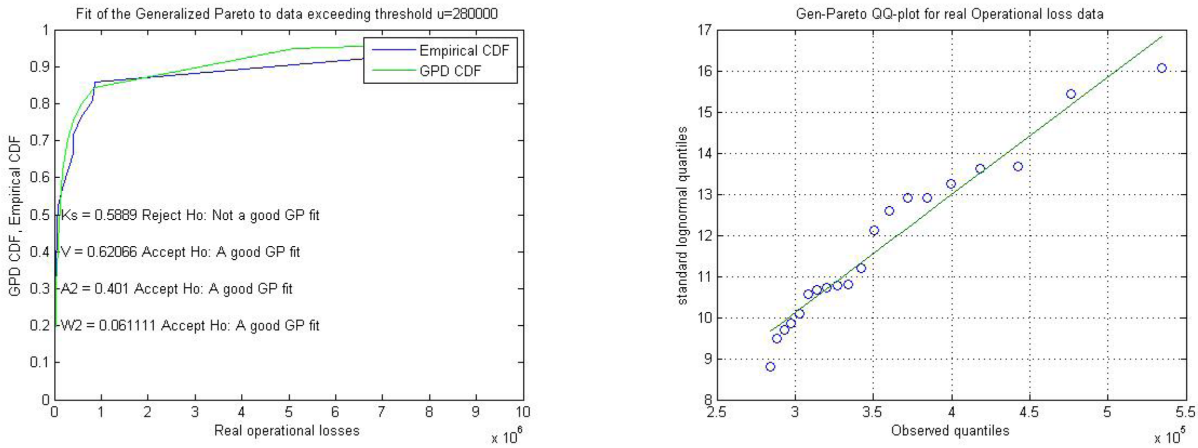

Figure 13: Peaks over threshold of SA retail bank's data

We see from the empirical fit, the GPD fits well to our data but deviates from it later, and we also see that the KS test rejected our fit. We do however get a good fit from our other three tests with the Cramer-von Misses test giving the smallest and best test with a test statistic of W2 = 0.061111. We see from the QQ-plot on the right that the data plots well along the fitted line, but we do see it deviate from the line as well. Overall we do have a good fit but the slight deviations can be accounted to the threshold chosen. We now look at the Value-at-Risk as estimated by the GPD under the POT (Figure 14).

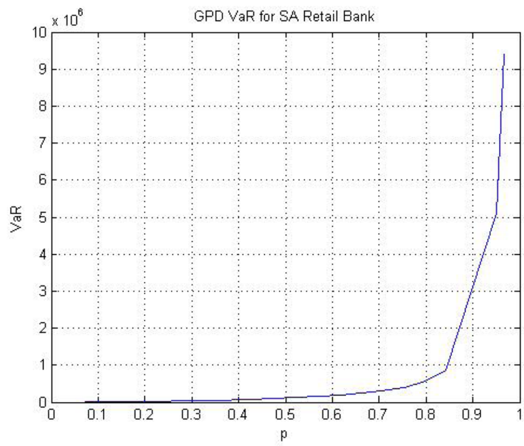

Figure 14: POT Value-at-Risk for SA Retail Bank data

We observe the 96.6% Value-at-Risk estimate at R9,389 000. Thus the bank should set aside this much to cater for extreme loss events exceeding the threshold chosen given that that threshold has been exceeded. Considering the Pareto estimate for low severity losses since the Pareto fit best to this data set together with this estimate for extreme events the bank is able to establish good cover for operational loss events of all types.

Conclusion

In this paper a Weibull(α, β), Pareto(α, β) and a longnormal(μ,σ) distributions with different parameterizations were fitted to two sets of real bank operational loss data one from a Spanish Saving bank and the other from a South African retail bank. Two graphical goodness-of-fit techniques, namely; the mean-excess plot and the quantile plots were used in judging whether a given distribution does fairly fit the data. To supplement the graphical goodness-of-fit test results, we used four common empirical distribution based goodness-of-fit tests, namely: the Kolmogorov-Smirnov, Cramer-von Misses, Anderson-Darling and the Kuiper test for goodness-of-fit to further asses the fitness of the hypothesized distribution on the data. Results from the simulation study suggest that, the Cramer-von Misses test provides the best fitness test statistic for the Lognormal and Weibull distribution. While the Kuiper test is best suited for judging the fitness in Pareto distribution.

After documenting which goodness-of-fit test best fit for what distribution all three considered loss distributions were fitted to the two datasets using the maximum likelihood method to estimate the parameters. The best fit distribution for each dataset was used to estimate the amount of capital that is needed for each bank to capitalize its exposure to operational risk. For the Spanish data, the Lognormal distribution provided the best fit and yielded a minimum capital estimation of €557 100.00 at a 99.8% confidence level. However, for the South African data, the Pareto distribution provided the best fit and yielded a minimum capital estimation of R9,669 000.00 at a 99.8% confidence level. It is worth noting that, in both cases none of the best fit distribution could achieve the 99.9% VaR as recommended in Basel II. Equally important, it is worth mentioning that, though the two best fit distributions, i.e. Lognormal and Pareto provided superior fitness to the two datasets respectively, their QQ-plots to some extend suggested evidence of under-fitting.

As a remedial approach, we further fitted a more heavy-tailed distribution (the Generalised Pareto) to both dataset and re-asses its fitness to both datasets. For each dataset a new capital was estimated. With the Spanish data, the GPD obtained a capital estimate of €407 100.00 at a 97.4%, while with the South African data an estimate of R9,389 000.00 at a 96.6% confidence level. In both case, high threshold values (u) of €150 000.00 and R =28000.00 as suggested by the mean excess function were used respectively. To this end, this paper therefore serves to suggest key fundamental techniques and procedures required in designing robust and effective operational risk capital allocation models. The authors believe the paper will greatly aid regulators and banks in emerging markets in effectively implementing the AMA as recommended in the Basel II accord.

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