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Mechanical Equations of Dynamic Systems on Weyl-Finsler Manifolds

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Abstract

Finsler geometry is an alternative approach to geometrization of fields and its fundamental idea can be traced of Riemann geometry so that Finsler geometry is a nearest generalization of Riemannian metric geometry. Finsler geometry was first applied in gravitational theory and this application lead to corrections to observational results predicted by general relativity. It's well known that there are various approaches to Finsler formulation of the general relativity and gauge field theories in the spacetime. Euler-Lagrange and Hamilton equations often used for modeling of mechanical systems. In this study, we will present Euler-Lagrange and Hamilton equations often used for modeling of mechanical systems of motion on Finsler manifolds such that they represent the dynamic state of the mechanical system. Also, implicit solutions of the differential equations found in this study are solved by Maple computation program.

Keywords: Weyl Manifold; Lagrangian; Hamiltonian; Conformal Structure; Dynamics Equation; Finsler Manifold

AMS: 30C20; 34B20; 58B20; 70H03; 70H05; 70E55

Introduction

A classical field theory explains the study of how one or more physical fields interact with matter which is used quantum and classical mechanics of physics branches. Also, a unified theory and it's the metric has been thought as the gravitational potential, as in general relativity, and the corresponding forms are thought as the electromagnetic potentials. Weyl's unified theory and it's the metric has been thought as the gravitational potential, as in general relativity, and the corresponding forms are thought as the electromagnetic potentials. Euler-(Lagrangian) and Hamiltonian models arise to be a very important tool and they present a simple method to describe the model for orbits of moving objects in the electromagnetic field.

There are many applications of differential geometry or mathematical physics. These applications are used in many areas of disciplines. There are many studies about Lagrangian and Hamiltonian dynamics, mechanics, formalisms, systems, equations and Finsler geometry [1]. Tekkoyun and Celik presented a new analogue of Euler-Lagrange and Hamilton equations on an almost Kähler model of a Finsler manifold [2]. Tekkoyun and Yayli presented generalized-quaternionic Kähler analogue of Lagrangian and Hamiltonian mechanical systems [3]. Wu established a relative volume comparison theorem for minimal volume form of Finsler manifolds under integral Ricci curvature bound [4]. Vries shown that the Hamiltonian and Lagrangian motion equations have a very simple interpretation in relativistic quantum mechanics [5]. Udriste and Neagu presented the basic properties of the scalar product along a curve and they investigated the variational formulae for the p-energy functional on a Finsler manifold [6]. Bercu showed the gradient method on Finsler manifold as to how to use the direction y for obtaining a suitable descent algorithm [7]. Szilasi and Tóth deduced consequences on vector field on the underlying manifold of a Finsler structure having one or two of the mentioned geometric properties [8]. Tayebi and Peyghan constructed a new class of Finsler metrics which is an extension of the class of Berwald metrics [9]. Bejancu constructed the transversal vector bundle of a coisotropic submanifolds of pseudo-Finsler manifold and obtained all structure equations of the degenerate immersion [10]. Lovas and Szilasi propose a new and complete proof of the theorem, discovered by Detlef Laugwitz: complete and connected finite dimensional Finsler manifolds admitting a proper homothety are Minkowski vector spaces [11]. Abate and Patrizio showed that a complex Finsler metric of constant holomorphic sectional curvature-4 satisfying the given symmetry condition on the curvature is necessarily the Kobayashi metric [12]. Almost contact Finsler structures on vector bundle are defined by Yaliniz and Caliskan and the condition of normality in terms of the Nijenhuis torsion $N\{\varphi\}$ of almost contact Finsler structure is obtained [13]. Vacaru considered some classes of exact solutions instring and Einstein gravity modelling Lagrange-Finsler structures with solitonic pp--waves and speculate on their physical meaning [14]. Kasap demonstrated Weyl-Euler-Lagrange and Weyl-Hamilton equations on $R\{n\}^{2n}$ which is a model of tangent manifolds of constant w-sectional curvature [15]. Kasap submitted Weyl-Euler-Lagrange equations of motion on flat manifold [16]. Miron and Anastasiei and their other colleagues there are summarized a number of results on almost Kähler-Lagrange-Finsler/Hamilton-Cartan geometries and generalizations with applications in mechanics [17].

Preliminaries

Definition 1: Let (M,F) be a connected n-dimensional Finsler manifold whose fundamental function verifies $F{:}TM \to R$ the following axioms:

(F1) $F(x,y)>0$; $\forall x\varepsilon M$, $\forall y\neq 0$.
(F2) $F(x,\lambda y)=|\lambda|F(x,y)$; $\forall \lambda\varepsilon R$, $\forall (x,y)\varepsilon TM$.
(F3) the fundamental tensor $g_{ij}(x,y)=(1/2)((\partial^2F^2)/(\partial y^i\partial y^j))$ is positive definite; $\forall x\varepsilon M$,

$(F1)\ F(x,y)>0;\forall x\varepsilon M,\forall y\neq 0.$
$(F2)\ F(x,\lambda y)=|\lambda|F(x,y);\forall\lambda\varepsilon R,\forall(x,y)\varepsilon TM.$
$(F3)$ *the fundamental tensor* $g_{ij}(x,y)=(1/2)\left(\left(\partial^2F^2\right)/\left(\partial y^i\partial y^j\right)\right)$ *is positive definite*; $\forall x\varepsilon M$,

(F4) F is C^c at every point $(x,y)\varepsilon TM$ with $y\neq 0$ and continuous at every $(x,0)\varepsilon TM$. Then, the absolute Finsler energy is $F^2(x,y)=g_{ij}(x,y)\,y^iy^j$.

Definition 2: $Let\, c: [a,b]\to M\ be\ a\ C^\infty$ regular curve on M. For any two vector fields $X(t)=X^i(t)\left(\partial/\left(\partial x^i\right)\right)|_{c(t)},\ Y(t)=Y^i(t)\left(\partial/\left(\partial x^i\right)\right)|c(t)$ along the curve c and the scalar product $g(X,Y)(c(t)=g_{ij}(c(t),\dot{C}(t))X^iY^j$ along the curve c.

Definition 3: Let M be a differentiable manifold of dimension (2n+1) and suppose J is a differentiable vector bundle isomorphism J:TM→TM such that J:TM→TM is an almost complex structure for TM.

Definition 4: An almost complex structure J on M assigns to each p∈M a linear map $J_p:T_pM\to T_pM$ that is smooth in p and satisfies $J_p{}^2=-Id$ for all p. The pair (M,J) is called an almost complex manifold. Any complex manifold M is also an almost complex manifold. In three dimensions, the vector from the origin to the point with Cartesian coordinates (x,y,z) can be written as [18].

$$r=x\vec{i}+y\vec{j}+z\vec{k}=x\left(\partial/\left(\partial x\right)\right)+y\left(\partial/\left(\partial y\right)\right)+z\left(\partial/\left(\partial z\right)\right). \tag{1}$$

Lemma 1: Let M be a smooth manifold. If M admits a complex structure A, then M admits an almost complex structure J. Let $\dim_C M=m$ and (Z,U) be any holomorphic chart inducing a coordinate frame $\partial x_1,\partial y_1,\ldots,\partial x_m,\partial y_m$. Then J is given locally as

$$Jp(\partial x_i|p)=\partial y_i\mid p,$$

$$Jp(\partial y_i\mid p)=-\partial x_i\mid p, \tag{2}$$

Where $1\leq i\leq m$ and p∈U [19]

Suppose M is a smooth manifold. Recall that a smooth curve in M is a smooth map $\gamma: I\to M,$ where I is an interval in R. For any a∈I, the tangent vector of γ at the point γ(a) is

$$\dot{\gamma}(a)=\left(\left(d\gamma\right)/\left(dt\right)\right)\left(a\right)=d_{\gamma(a)}\left(\left(d/\left(dt\right)\right)\right) \tag{3}$$

where d/dt is the standard coordinate tangent vector of R. Let X is a smooth vector field on M. We say that a smooth curve γ: I→M is an integral curve of X if for any t∈I,

$$\dot{\gamma}\left(t\right)=X_{\gamma(t)}. \tag{4}$$

Geodesics in Finsler Spaces

A Finsler manifold is a differentiable manifold together with the structure of an intrinsic quasisimetric space in which the length of any rectifiable curve γ:[a,b]→M is given by the length functional

$$L[\gamma]=\int_a^b F(\gamma(t),\dot{\gamma}(t))dt \tag{5}$$

Where $F(X,\dot{X})$ is a Minkowski norm on each tangent space T_xM. Finsler manifolds non-trivially generalize Riemannian manifolds in the sense that they are not necessarily infinitesimally Euclidean.

This means that the norm on each tangent space is not necessarily induced by an inner product (metric tensor). Due to the homogeneity of F the length (5) of a differentiable curve γ:[a,b]→M in M is invariant under positively oriented reparametrizations. A constant speed curve γ is a geodesic of a Finsler manifold if its short enough segments $\gamma|_{[c,d]}$ are length-minimizing in M from γ(c) to γ(d). Equivalently, γ is a geodesic if it is stationary for the energy functional

$$E[\gamma]=(1/2)\int_a^b F^2(\gamma(t),\dot{\gamma}(t))dt \tag{6}$$

in the sense that its functional derivative vanishes among differentiable curves γ:[a,b]→M with fixed endpoints γ(a)=x and γ(b)=y. A Finsler manifold is a differentiable manifold M together with a Finsler function F defined on the tangent bundle of M so that for all tangent vectors v. For each tangent vector v, the Hessian of F^2 at v is positive definite. Here the Hessian of F^2 at v is the symmetric bilinear form

$$g_v(X,Y)=(1/2)\left((\partial^2)/(\partial s\partial t)\right)\left[F(v+sX+tY)^2\right]|_{s=t=0}, \tag{7}$$

Also known as the fundamental tensor of F at v. Strong convexity of F^2 implies the subadditivity with a strict inequality if u/F(u)≠v/F(v). If F^2 is strongly convex, then F is a Murkowski norm on each tangent space.

Gauge Theory and Conformal Weyl Geometry

Hermann Weyl (1885-1955) made many fundamental and important contributions to physics. He is most famous for his 1929 discovery of quantum-mechanical phase invariance. Phase invariance known more properly as gauge invariance that it is symmetry and underlies all modern quantum theories. Throughout this section, M donates a smooth manifold of dimension n.

A conformal manifold is a differentiable manifold equipped with an equivalence class of (pseudo) Riemann metric tensors, in which two metrics g_2 and g_1 are equivalent if and only if

$$g_2=\Psi^2 g_1 \tag{8}$$

Where Ψ>0 is a smooth positive function. An equivalence class of such metrics is known as a conformal metric or conformal class and a manifold with a conformal structure (8) is called a conformal manifold [20]. A change in given by the global gauge transformation

$$\Psi(x)\to e^{i\lambda}\Psi(x) \tag{9}$$

Where λ is an arbitrary constant, would have absolutely no effect on the Lagrangian.
Action Lagrangians are invariant with respect to the replacement

$$\Psi(x)\to e^{i\lambda(x)}\Psi(x) \tag{10}$$

Can be made without changing anything essential. Where Ψ is a wave function and λ is an arbitrary function of space and time. (10) Is called a local gauge transformation

Weyl's gauge theory sprang from an even earlier (1918) theory in which Weyl demanded that Einstein's theory of general relativity should be invariant with respect to the similar replacement

$$g_{\mu\nu}(x)\to e^{\lambda(x)}g_{\mu\nu}(x) \tag{11}$$

Which we shall call a metric gauge transformation (11) and it has emerged effect of these transformations on Riemannian and non-Riemannian geometry. Also, it is remarkable that the Weyl tensor can be deduced by simply demanding that it be invariant with respect to this transformation. Weyl, using this gauge principle, was able to derive all of electrodynamics from a generalized Einstein-Maxwell Lagrangian. Weyl noticed that the magnitude of an arbitrary vector ξ^μ would undergo a rescaling under a gauge transformation given by

$$\begin{aligned}|\xi|^2&=g_{\mu\nu}\xi^\mu\xi^\nu,\\ |\xi'|^2&=g'_{\mu\nu}\xi^\mu\xi^\nu\\ &=e^{\lambda(x)}|\xi|^2,\end{aligned} \tag{12}$$

and he wondered if such a regauging would alter any essential physics. Today, the gauge principle is arguably the most powerful concept in all of modern physics. This gauge principle underlies all of the Yang-Mills theories and is a key component in string theory and its more recent variant, M theory [21].

Definition 5: Two Riemann metrics g_1 and g_2 on M are said to be conformally equivalent if there exists a smooth function f:M→R with

$$g_2 = e^f g_1 \tag{13}$$

In this case, $g_1 \sim g_2$. A pair (M,G), a conformal structure on M is an equivalence class G of Riemann metrics on M, is called a conformal structure.

Theorem 1: Let ∇ be a connection on M and g∈G a fixed metric. ∇ is compatible with (M,G) ⇔ there exists a 1-form ω with $\nabla_X g + \omega(X) g = 0$ [22, 23].

Definition 6: A compatible torsion-free connection is called a Weyl connection. The triple (M, G, ∇) is a Weyl structure. A Weyl manifold is a conformal manifold equipped with a torsion free connection preserving the conformal structure, called a Weyl connection.

Theorem 2: To each metric g∈G and 1-form ω, there corresponds a unique Weyl connection ∇ satisfying ∇Xg+ω(X)g=0. Here, ∇ is given by the equation [22].

$$g(\nabla_X Y,Z)=(1/2)\{X(g(Y,Z))+\omega(X)g(Y,Z)-g([X,Z],Y)+Y(g(X,Z))+\omega(Y)g(Z,X)-g([Y,X],Z)-Z(g(X,Y))-\omega(Z)g(X,Y)-([Z,Y],X)\} \tag{14}$$

(Proof see [22]).

Definitions 7: Define a function F:{ 1-forms on M }×G→{ Weyl connections } by F(g,ω)=∇ , where ∇ is the connection guaranteed by Theorem 2. We say that ∇ corresponds to (g,ω).

Proposition 1: F is subjective.

$$F(g,\omega)=F(e^f g,\eta) \text{ iff } \eta=\omega-df. \text{ So}$$

Proof: F is surjective by Theorem 3. In fact, Theorem 3 shows that given a compatible, torsion-free connection ∇ , then for every g∈G, there exists a 1-form ω with F(g,ω)=∇ .

Proposition 2:

$$F(e^f g)=F(g)-df, \tag{15}$$

Where G is a conformal structure. Note that a Riemann metric g and a one-form ω determine a Weyl structure, namely F: G→$\wedge^1$M where G is the equivalence class of g and $F(e^f g)=\omega-df$.

Proof: Suppose $F(g,\omega)=F(e^f g,\eta)=\nabla.$ We have

$$0=\nabla_X(e^f g)+\eta(X)e^f g = X(e^f)g+e^f\nabla_X g+\eta(X)\, e^f g \;= df(X)e^f g+e^f\nabla_X g+\eta(X)e^f g. \tag{16}$$

Therefore $\nabla_X g=-(df(X)+\eta(X))$. On the other hand, $\nabla_X g+\omega(X)g=0$ and $\omega=\eta+df$. Conversely, suppose $\eta=\omega-df$. Set $\nabla=F(g,\omega)$. To show $\nabla=F(e^f g,\eta)$, it suffices, by the uniqueness of Theorem 2, to show

$$\nabla_X(e^f g)+\eta(X)e^f g=0. \tag{17}$$

Let's show the truth of this statement. $\nabla_X(e^f g)=e^f df(X)g+e^f\nabla_X g \text{ and } \eta(X)=\omega(X)-df(X).$ $\nabla_X(e^f g)+\eta(X)e^f g=e^f df(X)g+e^f\nabla_X g+(\omega(X)-df(X))e^f g$

$$=e^f df(X)g+e^f\nabla_X g+\omega(X)e^f g-df(X)e^f g$$

$$=e^f(\nabla_X g+\omega(X)g)=0. \tag{18}$$

Such a geometric structure was introduced by Weyl (in 1922) in an attempt to unify gravity with electromagnetism [24]. Let (M,g) is conformally flat if for each point x in M, there exists a neighborhood U of x and a smooth function f defined on U such that (U, e^{2f}g) is flat. The function f need not be defined on all of M [25].

Theorem 3: Let ∇ be a torsion free connection on the tangent bundle of M and m≥6. If (M, g, ∇ , J) is a Kähler-Weyl structure, then the associated Weyl structure is trivial, i.e. there is a conformally equivalent metric

$$g_1=e^{2f}g, \tag{19}$$

So that (M,g_1,J) is Kähler and so that $\nabla = \nabla g_1$ [proof see 26].

Weyl transformation is a local rescaling of the metric tensor:

$$g_{ab}(x) \to e^{-2\omega(x)} g_{ab}(x), \tag{20}$$

which produces another metric in the same conformal class. A theory or an expression invariant under this transformation is called conformally invariant, or is said to possess Weyl symmetry. The Weyl symmetry is an important symmetry in conformal field theory [27]. Weyl curvature tensor is a measure of the curvature of spacetime or a pseudo-Riemannian manifold. Like the Riemannian curvature tensor, the Weyl tensor expresses the tidal force that a object feels when moving along a geodesic.

Finsler Geometry

In this section, we recall some structures given in [28, 29].

Definitions 8: A Finsler manifold (space) is a pair $F^n = (M, F(x,y))$ where M is a real n-dimensional differential manifold and F:TM→R a scalar function which satisfy the following axioms:
i) F is a differentiable function on the manifold $\widetilde{TM} = TM \setminus \{0\}$ and F is continuous on the null section of the projection π:TM→M.
ii) F is positive function.
iii) F is positively 1-homogeneous on the fibers of tangent bundle TM.
iv) The Hessian of F^2 with elements.

$$g_{ij}(x,y) = (1/2)\left((\partial^2 F^2)/(\partial y^i \partial y^j)\right) \tag{21}$$

is positively defined on $\tilde{T}\tilde{M}$. Where gij is a covariant symmetric of 2 order distinguished tensor field (d-tensor field) defined on the manifold $\tilde{T}\tilde{M}$ The function F(x,y) is called fundamental function and the d-tensor field gij is called fundamental (or metric) tensor of the Finsler space $F^n = (M, F(x,y))$.

A Finsler space $F^n=(M,F)$ can be thought as an almost Kähler space on the manifold $\tilde{T}\tilde{M} = TM \setminus \{0\}$, called the geometrical model of the Finsler space F^n. If we consider the Cartan nonlinear connection N^i_j of the Finsler space $F^n = (M,F)$, then we can respectively define almost complex structures F and F* on TM and T*M by:

$$(1) \quad F^*(dx^i) = -\delta y^i, \quad F^*(\delta y^i) = dx^i.$$

$$(2) \quad F\left(\delta/(\delta x^i)\right) = -\partial/(\partial y^i), \quad F\left(\partial/(\partial y^i)\right) = \delta/(\delta x^i). \tag{22}$$

It is easy to see that F is well defined on TM, $F^2=-I$ and it is determined only by the fundamental function F of the Finsler space F^n. Also,

$$\delta/(\delta x^i) = \delta/(\delta x^i) - N^i_j(x,y)(\delta/(\delta y^i)) \in TM,$$

$$\left(\partial/(\partial x^i)\right)^H = \delta/(\delta x^i),$$

$$\delta y^i = dx^i + N^i_j(x,y)dy^i \in T^*M$$

$$(dy^i)^H = \delta y^i. \tag{23}$$

Let $(dx^i,\delta y^i)$ be the dual basis of the adapted basis $\delta/(\delta x^i)$, $\partial/(\partial y^i)$). Then, the Sasaki-Matsumoto lift of the fundamental tensor g_{ij} can be introduced as follows:

$$G = g_{ij} dx^i \otimes dx^j + g_{ij} \delta y^i \otimes \delta yj. \tag{24}$$

Consequently, G is a Riemann metric on determined only by the fundamental function F of the Finsler space F^n and the horizontal and vertical distributions are orthogonal with respect to it.

i) The pair (G,F) is an almost Hermitian structure on $H^{2n} = (;G,F) H^2 = (;G,F)$.
ii) The almost symplectic 2-form associated to the almost Hermitian structure (G,F) is

$$\theta = g_{ij}(x,y)\delta y^i \otimes dx^j. \tag{25}$$

iii) The space $H^{2n}=(TM;G,F)$ is an almost Kähler space, constructed only by means of the fundamental function F of the Finsler space F^n.

The space $H^{2n}=\tilde{T}\tilde{M}$ (;G,F) is called the almost Kähler model of the Finsler space F^n.

Remarking that the following tensor field on $\tilde{T}\tilde{M}$,

$$\dot{G} = g_{ij}x,y)dx^i \otimes dx^j + \left((a^2)/(\| y \|^2)\right) g_{ij}(x,y)\delta y^i \otimes \delta y^j, \quad \forall (x,y) \in TM. \tag{26}$$

(26) Is the homogeneous lift to TM of the fundamental tensor field g_{ij} of a Finsler space F^n, where a>0 is a constant, imposed by applications (in order to preserve the physical dimensions of the components of $\dot{G}$ and where $\|y\|^2$ is the square of the norm of the Liouville vector field:

$$\| y \|^2 = g_{ij}(x,y) y^i y^j = y_i y^i = F^2(x,y), \tag{27}$$

and for $y_i = g_{ij}(x,y)y^j = (1/2)\left((\partial F^2)/(\partial y^i)\right)$.

Let us prove that the almost complex structure F, defined by (22) does not preserve the property of homogeneity of the vector fields. Indeed, it applies the 1-homogeneous vector fields $\left(\delta/(\delta x^i)\right), (i=1,n)$ onto the homogenous vector fields $\partial/(\partial y^i)$, $(i=1,\ldots,n)$. We can eliminate this inconvenient by defining a new kind of almost complex structure F: $\chi(TM) \to \chi(TM)$,
setting:

$$\dot{F}\left(\delta/(\delta x^i)\right) = -\left((\| y \|)/a\right)\left(\partial/(\partial y^i)\right)$$

$$\dot{F}\left(\partial/(\partial y^i)\right)) = \left(a/(\| y \|)\right)\left(\delta/(\delta x^i)\right),$$

$$\dot{F}^*\left(dx^i\right) = -\left((\| y \|)/a\right)\delta y^i,$$

$$\dot{F}^*\left(\delta y^i\right) = \left(a/(\| y \|)\right)dx^i. \tag{28}$$

Proposition 3: If we extend the equation (28) by means of conformal structure [23], (19, 20), we can give equations as follows:

$$\dot{F}_w\left(\delta/(\delta x^i)\right) = -\left((\| y \|)/a\right)e^{-2\lambda}\left(\partial/(\partial y^i)\right),$$

$$\dot{F}_w\left(\partial/(\partial y^i)\right) = \left(a/(\| y \|)\right)e^{2\lambda}\left(\delta/(\delta x^i)\right), \tag{29}$$

Such that are base structures. Where $\dot{F}_W$ is a conformal complex structure to be similar to an integrable almost complex structure $\dot{F}$ given in (28). Similarly, $\dot{F}^*$w is the dual of $\dot{F}_w$ structure and are defined the following as:

$$\dot{F}^*_w\left(dx^i\right) = -\left((\backslash y \backslash)/a\right)e^{-2\lambda}\delta y^i,$$

$$\dot{F}^*_w\left(\delta y^i\right) = \left(a/(\backslash y \backslash)\right)e^{2\lambda}dx^i, \tag{30}$$

such that is base conformal complex structure.

Lagrange Dynamics Equations

These dynamic equations are illustrated as follows [33-35].

Lemma 2: The closed 2-form on a vector field and 1-form reduction function on the phase space defined of a mechanical system is equal to the differential of the energy function 1-form of the Lagrangian mechanical systems [30].

Definitions 9: Let M be an n-dimensional manifold and TM its tangent bundle with canonical projection τ_M:TM→M. TM is called the phase space of velocities of the base manifold M. Let L:TM→R be a differentiable function on TM called the Lagrangian function. Here, L=K-P such that K is the kinetic energy and P is the potential energy of a mechanical system. In the problem of a mass on the end of a spring, $T = m\dot{x}^2/2$ *and* $V = kx^2/2$, so we have $L = m\dot{x}^2/2 - kx^2/2$. We consider the closed 2-form and base space (J) on TM given by $\Phi_L = -d\mathbf{d}_\mathbf{J}L = -d(J(\mathbf{d}))$. Consider the equation

$$i_\xi \Phi_L = dE_L. \tag{31}$$

Where i_ξ is reduction function and $i_\xi \Phi_L = \Phi_L(\xi)$ is defined in the form. Then ξ is a vector field, we shall see that (31) under a certain condition on ξ is the intrinsically expression of the Euler-Lagrange equations of motion. This equation (31) is named as Lagrange dynamical equation. We shall see that for motion in a potential, EL=VL-L is an energy function and $V = J\xi$ a Liouville vector field. Here dE_L denotes the differential of E. The triple (TM, Φ_L, ξ) is known as Lagrangian system on the tangent bundle TM. If it is continued the operations on (31) for any coordinate system then infinite dimension Lagrange's equation is obtained the form below. The equations of motion in Lagrangian mechanics are the Lagrange equations of the second kind, also known as the Euler-Lagrange equations;

$$\left(\partial / (\partial t)\right)((\partial L)/(\partial \dot{x})) = (\partial L)/(\partial x). \tag{32}$$

Newton's second law of motion for the mechanical problem is F=ma. Where, the vector sum of the external forces F on an object is equal to the mass m of that object multiplied by the acceleration vector a of the object.

Proposition 4: We have $(\partial L)/(\partial \dot{x}) = mx \ and \ (\partial L)/(\partial x) = -kx$, so eq. (32) give $m\ddot{x} = -kx$ sThe Euler-Lagrange equation, eq. (32), gives $m\ddot{x} = -(dV)/(dx)$. In a two-dimensional setup written in terms of Cartesian coordinates, the potential takes the form V(x,y), so the Lagrangian is $L = (1/2)m_i(\dot{x}^2 + \dot{y}^2) - V(x,y)$. So, the three Euler-Lagrange equations may be combined into the vector statement $m\ddot{x} = -\nabla V$.

Hamilton Dynamics Equations

Lagrangian formulation is an important springboard from which to develop another useful formulation of classical mechanics known as the Hamiltonian formulation. The Hamiltonian of a system is defined to be the sum of the kinetic (K) and potential energies (P) expressed as a function of positions and their conjugate momenta.

Definitions 10: Let M is the configuration manifold and its cotangent manifold T^*M. By a symplectic form we mean a 2-form Φ on T^*M such that:
(i) Φ is closed, that is $d\Phi = 0$;
(ii) For each $z \in T^*M$, Φ: $T^*M \times T^*M \to R$ is weakly nondegenerate.

If Φ_z in (ii) is nondegenerate, we speak of a strong symplectic form. If (ii) is dropped we refer to Φ as a presymplectic form. Let (T^*M, Φ) be a symplectic manifold. A vector field $X_H : T^*M \to T^*M$ is called Hamiltonian if there is a C^1 function $H : T^*M \to R$ such that dynamical equation is determined by

$$iX_H \Phi = dH. \tag{33}$$

We can say that XH is locally Hamiltonian vector field if $iX_H\Phi$ is closed and where Φ shows the canonical symplectic form so that $\Phi = -d\Omega$, $\Omega = J^*(\omega)$, J^* a dual of J, ω a 1-form on T^*M. The trio (T^*M, Φ, X_H) is named Hamiltonian system which it is defined on the cotangent bundle T^*M [1]. The vector field X on T^*M given by $i_X\omega = dH$ is called the geodesic flow of the metric g. If $\gamma : (a,b) \to T^*M$ is an integral curve of the geodesic flow, then the curve $p(\gamma)$ in M is called a geodesic. Recall from elementary physics that momentum of a particle, p_i, is defined in terms of its velocity $\dot{q}_i by\ p_i = m_i \dot{q}_i$. In fact, the more general definition of conjugate momentum, valid for any set of coordinates, is given in terms of the Lagrangian: $p_i = (\partial L)/(\partial \dot{q}_i)$. Note that these two definitions are equivalent for Cartesian variables. In terms of Cartesian momenta, the kinetic energy is given by $K = \sum_{i=1}^n p_i^2 / 2m_i$. Then, the Hamiltonian, which is defined to be the sum, H=K+P, expressed as a function of positions and momenta, will be given by

$$H(p,q) = \sum_{i=1}^n p_i^2 / 2m_i + P(q_1, \ldots, qn) \tag{34}$$

Where $p = p_1, \ldots, p_n$. The function H is equal to the total energy of the system. In terms of the Hamiltonian, the equations of motion of a system are given by Hamilton's equations:

$$\dot{q}_i = (\partial H)/(\partial p_i),$$

$$\dot{p}_i = -(\partial H)/(\partial q_i) \tag{35}$$

[31]. The solution of Hamilton's equations of motion will yield a trajectory in terms of positions and momenta as functions of time. Hamilton's equations can be easily shown to be equivalent to Newton's equations, and, like the Lagrangian formulation, Hamilton's equations can be used to determine the equations of motion of a system in any set of coordinates.

Lagrangian Dynamics

Using Lemma 2, we obtain conformal Euler-Lagrange equations for quantum and classical mechanics on the almost Kähler model $\dot{H}^{2n} = (\tilde{T}\tilde{M}; \dot{G}, \dot{F}_w)$ of the Finsler space F^n.

Proposition 5: Let $\dot{F}_w$ be a complex structure on the almost Kähler model $\dot{H}^{2n}$ of the Finsler space F^n, and $\{x^i, y^i\}$ be its coordinate functions. Let semispray be the vector field ξ determined by

$$\xi = X^i\left(\delta/\left(\delta x^i\right)\right) + Y^i\left(\partial/\left(\partial y^i\right)\right), \tag{36}$$

Where $X^i = y^i = \dot{x}^i,\ Y^i = \dot{y}^i = \ddot{X}^i$ and the dot indicates the derivative with respect to time t. The vector field defined by

$$V = \dot{F}_w(\xi) = -X^i\left(\left(\backslash y \backslash\right)/a\right)e^{-2\lambda}\left(\partial/\left(\partial y^i\right)\right) + Y^i\left(a/\left(\backslash y \backslash\right)\right)e^{2\lambda}\left(\delta/\left(\delta x^i\right)\right) \tag{37}$$

is named Liouville vector field on the almost Kähler model H^{2n} of the Finsler space F^n. The maps given by K, $P: M \to R \text{ such that } K = (1/2)m_i(\dot{x}^2 + \dot{y}^2),\ P = m_i gh$ are said to be the kinetic energy and the potential energy of the system, respectively. Here m_i, g and h stand for mass of a mechanical system having m particles, the gravity acceleration and distance to the origin of a mechanical system on the almost Kähler model $\dot{H}^{2n}$ of the Finsler space F^n, respectively. Then L:M→R is a map that satisfies the conditions; i) L=K-P is a Lagrangian function, ii) the function determined by $E_L = V\dot{F}_w(L) - L,$ is energy function. The function $i\dot{F}_w$ induced by $\dot{F}_w$ and denoted by

$$i\dot{F}_w\omega\left(X_1, X_2, \ldots, X_r\right) = \sum_{i=1}^{r}\omega\left(X_1, \ldots, F_w X_i, \ldots, X_r\right), \tag{38}$$

is called vertical derivation, where $\omega \in \wedge^r M,\ X_i \in \chi(M)$. The vertical differentiation $d\dot{F}$ is given by $d\dot{F}_w = [i\dot{F}_w, d] = i\dot{F}_w d - di\dot{F}_w,$ where d is the usual exterior derivation. For $\dot{F}_w$, the closed Kähler form is the closed 2-form given by $\Phi_L = -\ dd\dot{F}_w L$ such that

$$d\dot{F}_w = -\left(\left(\backslash y \backslash\right)/a\right)e^{-2\lambda}\left(\partial/\left(\partial y^i\right)\right)dx^i + \left(a/\left(\backslash y \backslash\right)\right)e^{2\lambda}\left(\delta/\left(\delta x^i\right)\right)\delta y^i,\ \ F(\tilde{T}\tilde{M}) \to \wedge\tilde{T}\tilde{M}. \tag{39}$$

Then we have

$$\begin{aligned}
\Phi_L &= -dd\dot{F}_w L \\
&= [\left(\left(\backslash y \backslash\right)/a\right)2e^{-2\lambda}\left(\left(\partial\lambda\right)/\left(\partial x^i\right)\right)\left(\left(\partial L\right)/\left(\partial y^i\right)\right)dx^j \wedge dx^i - \left(\left(\backslash y \backslash\right)/a\right)e^{2\lambda}\left(\left(\partial^2 L\right)/\left(\partial x^j\partial y^i\right)\right)dx^j \wedge dx^i \\
&+ \left(a/\left(\backslash y \backslash\right)\right)2e^{2\lambda}\left(\left(\partial\lambda\right)/\left(\partial x^j\right)\right)\left(\left(\delta L\right)/\left(\delta x^i\right)\right)dy^i \wedge dx^j + \left(a/\left(\backslash y \backslash\right)\right)e^{2\lambda}\left(\left(\partial L\delta L\right)/\left(\delta x^j\partial x^i\right)\right)dy^i \wedge dx^j \\
&+ \left(\left(\backslash y \backslash\right)/a\right)2e^{-2\lambda}\left(\left(\partial\quad\right)/\left(\partial y^j\right)\right)\left(\left(\partial L\right)/\left(\partial y^i\right)\right)dx^i \wedge dy^j - \left(\left(\backslash y \backslash\right)/a\right)e^{-2\lambda}\left(\left(\partial^2 L\right)/\left(\partial y^j\partial y^i\right)\right)dx^i \wedge dy^j \\
&+ \left(a/\left(\backslash y \backslash\right)\right)2e^{2\lambda}\left(\left(\partial\lambda\right)/\left(\partial y^j\right)\right)\left(\left(\delta L\right)/\left(\delta x^i\right)\right)dy^i \wedge dy^j + \left(a/\left(\backslash y \backslash\right)\right)e^{2\lambda}\left(\left(\partial L\delta L\right)/\left(\partial y^j\delta x^i\right)\right)dy^i \wedge dy^j].
\end{aligned} \tag{40}$$

Let ξ be the second order differential equation (semispray) given by (36). Then we calculate

$$\begin{aligned}
i_\xi\Phi_L &= \Phi_L(\xi) \\
&= [\left(\left(\backslash y \backslash\right)/a\right)2e^{-2\lambda}\left(\left(\partial\lambda\right)/\left(\partial x^j\right)\right)\left(\left(\partial L\right)/\left(\partial y^i\right)\right)X^j dx^i - \left(\left(\backslash y \backslash\right)/a\right)2e^{-2\lambda}\left(\left(\partial\lambda\right)/\left(\partial x^j\right)\right)\left(\left(\partial L\right)/\left(\partial y^i\right)\right)X^i dx^j \\
&- \left(\left(\backslash y \backslash\right)/a\right)e^{-2\lambda}\left(\left(\partial^2 L\right)/\left(\partial x^j\partial y^i\right)\right)X^j dx^i + \left(\left(\backslash y \backslash\right)/a\right)e^{-2\lambda}\left(\left(\partial^2 L\right)/\left(\partial x^j\partial y^i\right)\right)X^i dx^j \\
&- \left(a/\left(\backslash y \backslash\right)\right)2e^{2\lambda}\left(\left(\partial\lambda\right)/\left(\partial x^j\right)\right)\left(\left(\delta L\right)/\left(\delta x^i\right)\right)X^j dy^i - \left(a/\left(\backslash y \backslash\right)\right)e^{2\lambda}\left(\left(d\delta L\right)/\left(\delta x^j\partial x^i\right)\right)X^j dy^i \\
&+ \left(\left(\backslash y \backslash\right)/a\right)2e^{-2\lambda}\left(\left(\partial\lambda\right)/\left(\partial y^j\right)\right)\left(\left(\partial L\right)/\left(\partial y^i\right)\right)X^i dy^j - \left(\left(\backslash y \backslash\right)/a\right)e^{-2\lambda}\left(\left(\partial^2 L\right)/\left(\partial y^j\partial y^i\right)\right)X^i dy^j \\
&+ \left(a/\left(\backslash y \backslash\right)\right)2e^{2\lambda}\left(\left(\partial\lambda\right)/\left(\partial x^j\right)\right)\left(\left(\delta L\right)/\left(\delta x^i\right)\right)Y^i dx^j + \left(a/\left(\backslash y \backslash\right)\right)2e^{2\lambda}\left(\left(d\delta L\right)/\left(\partial x^j\delta x^i\right)\right)Y^i dx^j \\
&- \left(\left(\backslash y \backslash\right)/a\right)2e^{-2\lambda}\left(\left(\partial\lambda\right)/\left(\partial y^j\right)\right)\left(\left(\partial L\right)/\left(\partial y^i\right)\right)Y^j dx^i + \left(\left(\backslash y \backslash\right)/a\right)e^{-2\lambda}\left(\left(\partial^2 L\right)/\left(\partial y^j\partial y^i\right)\right)Y^j dx^i \\
&+ \left(a/\left(\backslash y \backslash\right)\right)2e^{2\lambda}\left(\left(\partial\lambda\right)/\left(\partial y^j\right)\right)\left(\left(\delta L\right)/\left(\delta x^i\right)\right)Y^i dy^j - \left(a/\left(\backslash y \backslash\right)\right)2e^{2\lambda}\left(\left(\partial\lambda\right)/\left(\partial y^j\right)\right)\left(\left(\delta L\right)/\left(\delta x^i\right)\right)Y^j dy^i \\
&+ \left(a/\left(\backslash y \backslash\right)\right)e^{2\lambda}\left(\left(d\delta L\right)/\left(\partial y^j\delta x^i\right)\right)Y^i dy^j - \left(a/\left(\backslash y \backslash\right)\right)e^{2\lambda}\left(\left(\partial\delta L\right)/\left(\partial y^j\delta x^i\right)\right)Y^j dy^i].
\end{aligned} \tag{41}$$

Energy function and its differential are

$$E_L = -X^i\left(\left(\backslash y \backslash\right)/a\right)e^{-2\lambda}\left(\left(\partial L\right)/\left(\partial y^i\right)\right) + Y^i\left(a/\left(\backslash y \backslash\right)\right)e^{2\lambda}\left(\left(\delta L\right)/\left(\delta x^i\right)\right) - L. \tag{42}$$

and

$$\begin{aligned}
dE_L &= X^i\left(\left(\backslash y \backslash\right)/a\right)2e^{-2\lambda}\left(\left(\partial\lambda\right)/\left(\partial x^j\right)\right)\left(\left(\partial L\right)/\left(\partial y^i\right)\right)dx^j - X^i\left(\left(\backslash y \backslash\right)/a\right)e^{-2\lambda}\left(\left(\partial^2 L\right)/\left(\partial x^j\partial y^i\right)\right)dx^j \\
&+ Y^i\left(a/\left(\backslash y \backslash\right)\right)2e^{2\lambda}\left(\left(\partial\lambda\right)/\left(\partial x^j\right)\right)\left(\left(\delta L\right)/\left(\delta x^i\right)\right)dx^j + Y^i\left(a/\left(\backslash y \backslash\right)\right)e^{2\lambda}\left(\left(\delta L\partial L\right)/\left(\delta x^j\partial x^i\right)\right)dx^j - \left(\left(\delta L\right)/\left(\delta x^i\right)\right)dx^i
\end{aligned}$$

$$+X^i\left(\left(\backslash y\backslash\right)/a\right)2e^{2\lambda}\left(\left(\partial\lambda\right)/\left(\partial y^i\right)\right)\left(\left(\partial L\right)/\left(\partial y^i\right)\right)dy^i - X^i\left(\left(\backslash y\backslash\right)/a\right)2e^{-2\lambda}\left(\left(\partial^2 L\right)/\left(\partial y^i\partial y^i\right)\right)dy^i$$

$$+Y^i\left(a/\left(\backslash y\backslash\right)\right)2e^{2\lambda}\left(\left(\partial\lambda\right)/\left(\partial y^i\right)\right)\left(\left(\delta L\right)/\left(\delta x^i\right)\right)dy^i + Y^i\left(a/\left(\backslash y\backslash\right)\right)e^{2\lambda}\left(\left(\delta L\partial L\right)/\left(\partial y^i\delta x^i\right)\right)dy^i - \left(\left(\partial L\right)/\left(\partial y^i\right)\right)dy^i. \quad (43)$$

Using $i_\xi\Phi_L = dE_L$, we find the expressions as follows:

$$-\left(\left(\backslash y\backslash\right)/a\right)2e^{-2\lambda}\left(\left(\partial\lambda\right)/\left(\partial x^i\right)\right)\left(\left(\partial L\right)/\left(\partial y^i\right)\right)X^i dx^i + \left(\left(\backslash y\backslash\right)/a\right)e^{-2\lambda}\left(\left(\partial^2 L\right)/\left(\partial x^i\partial y^i\right)\right)X^i dx^i$$

$$\left(\left(\backslash y\backslash\right)/a\right)2e^{-2\lambda}\left(\left(\partial\lambda\right)/\left(\partial y^i\right)\right)\left(\left(\partial L\right)/\left(\partial y^i\right)\right)Y^i dx^i + \left(\left(\backslash y\backslash\right)/a\right)e^{-2\lambda}\left(\left(\partial^2 L\right)/\left(\partial y^i\partial y^i\right)\right)Y^i dx^i$$

$$\left(a/\left(\backslash y\backslash\right)\right)2e^{2\lambda}\left(\left(\partial\lambda\right)/\left(\partial x^i\right)\right)\left(\left(\delta L\right)/\left(\delta x^i\right)\right)X^i dy^i - \left(a/\left(\backslash y\backslash\right)\right)e^{2\lambda}\left(\left(d\delta L\right)/\left(\delta x^i\partial x^i\right)\right)X^i dy^i$$

$$\left(a/\left(\backslash y\backslash\right)\right)2e^{2\lambda}\left(\left(\partial\lambda\right)/\left(\partial y^i\right)\right)\left(\left(\delta L\right)/\left(\delta x^i\right)\right)Y^i dy^i - \left(a/\left(\backslash y\backslash\right)\right)e^{2\lambda}\left(\left(d\delta L\right)/\left(\partial y^i\delta x^i\right)\right)Y^i dy^i$$

$$\left(\left(\delta L\right)/\left(\delta x^i\right)\right)dx^i - \left(\left(\partial L\right)/\left(\partial y^i\right)\right)dy^i, \quad (44)$$

And then

$$-\left(\left(\backslash y\backslash\right)/a\right)2e^{-2\lambda}\left(\partial/\left(\partial t\right)\right)\left(\lambda\right)\left(\left(\left(\partial L\right)/\left(\partial y^i\right)\right)\right) + \left(\left(\backslash y\backslash\right)/a\right)e^{-2\lambda}\left(\partial/\left(\partial t\right)\right)\left(\left(\left(\partial L\right)/\left(\partial y^i\right)\right)\right) + \left(\delta L\right)/\left(\delta x^i\right) = 0,$$

$$-\left(a/\left(\backslash y\backslash\right)\right)2e^{2\lambda}\left(\partial/\left(\partial t\right)\right)\left(\lambda\right)\left(\left(\left(\delta L\right)/\left(\delta x^i\right)\right)\right) - \left(a/\left(\backslash y\backslash\right)\right)e^{2\lambda}\left(\partial/\left(\partial t\right)\right)\left(\left(\left(\delta L\right)/\left(\delta x^i\right)\right)\right) + \left(\partial L\right)/\left(\partial y^i\right) = 0. \quad (45)$$

We have the equations with integral curve definition $\xi(\alpha)=\alpha(t)=(\partial/(\partial t))(\alpha)$,

$$\left(\left(\backslash y\backslash\right)/a\right)\left(\partial/\left(\partial t\right)\right)\left(e^{-2\lambda}\left(\left(\partial L\right)/\left(\partial y^i\right)\right)\right) + \left(\delta L\right)/\left(\delta x^i\right) = 0,$$

$$\left(a/\left(\backslash y\backslash\right)\right)\left(\partial/\left(\partial t\right)\right)\left(e^{2\lambda}\left(\left(\delta L\right)/\left(\delta x^i\right)\right)\right) - \left(\partial L\right)/\left(\partial y^i\right) = 0, \quad (46)$$

such that the equations calculated in (46) are named conformal Weyl-Euler-Lagrange equations constructed on the almost Kähler model $\dot{H}^{2n}$ of the Finsler space F^n and thus the triple ($\dot{H}^{2n}$,ΦL,ξ) is called a Weyl-Euler-Lagrange mechanical system on the almost Kähler model $\dot{H}^{2n}$ of the Finsler space F^n. Hence, if the above equations (46) λ=0 is selected, the equations [2] are obtained.

$$\left(\left(\backslash y\backslash\right)/a\right)\left(\partial/\left(\partial t\right)\right)\left(\left(\partial L\right)/\left(\partial y^i\right)\right) + \left(\left(\delta L\right)/\left(\delta x^i\right)\right) = 0,$$

$$\left(a/\left(\backslash y\backslash\right)\right)\left(\partial/\left(\partial t\right)\right)\left(\left(\delta L\right)/\left(\delta x^i\right)\right) - \left(\left(\partial L\right)/\left(\partial y^i\right)\right) = 0. \quad (47)$$

Hamiltonian Dynamics

Using Lemma 2, we present conformal Hamilton equations and Hamiltonian mechanical systems for quantum and classical mechanics constructed on the almost Kähler model $\dot{H}^{2n}=((\tilde{T}\tilde{M});\dot{G},\dot{F}_w^*)$ of the Finsler space F^n.

Proposition 6: Let ω be set a 1-form

$$\omega = \left(\left(a^2\right)/\left(\backslash y\backslash^2\right)\right)x^i\delta x^i + y^i dy^i. \quad (48)$$

Then we have the Liouville form

$$\Omega = \dot{F}_w^*(\omega) = \dot{F}_w^*(((a^2)/(\backslash y\backslash^2))x^i\delta x^i + y^i dy^i) = -(a/(\backslash y\backslash))x^i e - 2^\lambda dy^i + (a/(\backslash y\backslash))y^i e^{2\lambda}\delta xi, \quad (49)$$

And the closed form

$$\varphi = -d\Omega$$

$$=(1/2)[(a/(\backslash y\backslash))x^i 2e^{-2\lambda}\left(\left(\partial\lambda\right)/\left(\partial x^j\right)\right)dx^j\wedge dy^i - \left(a/\left(\backslash y\backslash\right)\right)e^{-2\lambda}\left(\left(\partial x^i\right)/\left(\partial x^j\right)\right)dx^j\wedge dy^i$$

$$+\left(a/\left(\backslash y\backslash\right)\right)y^i 2e^{2\lambda}\left(\left(\partial\lambda\right)/\left(\partial x^j\right)\right)dx^j\wedge\delta x^i + \left(a/\left(\backslash y\backslash\right)\right)e^{2\lambda}\left(\left(\partial y^i\right)/\left(\partial x^j\right)\right)dx^j\wedge\delta x^i$$

$$+\left(a/\left(\backslash y\backslash\right)\right)x^i 2e^{-2\lambda}\left(\left(\partial\lambda\right)/\left(\partial y^j\right)\right)dy^j\wedge dy^i - \left(a/\left(\backslash y\backslash\right)\right)e^{-2\lambda}\left(\left(\partial x^i\right)/\left(\partial y^j\right)\right)dy^j\wedge dy^i$$

$$+\left(a/\left(\backslash y\backslash\right)\right)y^{i}2e^{2\lambda}\left(\left(\partial\lambda\right)/\left(\partial y^{j}\right)\right)dy^{j}\wedge\delta x^{i}+\left(a/\left(\backslash y\backslash\right)\right)e^{2\lambda}\left(\left(\partial y^{i}\right)/\left(\partial y^{j}\right)\right)dy^{j}\wedge\delta x^{i}]. \quad (50)$$

Proposition 7. Let X_H be take Hamiltonian vector field as follows:

$$X_H = X^{i}\left(\partial/\left(\partial x^{i}\right)\right)+Y^{i}\left(\delta/\left(\delta y^{i}\right)\right). \quad (51)$$

Then we find

$$\begin{aligned}
&i_{XH}\Phi=\Phi\left(X_H\right)\\
&=(1/2)\left[-X^{i}\left(a/\left(\backslash y\backslash\right)\right)2e^{-2\lambda}\left(\partial\lambda\right)/\left(\partial x^{j}\right)\right)x^{i}dy^{i}+X^{i}\left(a/\left(\backslash y\backslash\right)\right)e^{-2\lambda}dy^{i}\\
&+X^{i}\left(a/\left(\backslash y\backslash\right)\right)y^{i}2e^{2\lambda}\left(\left(\partial\lambda\right)/\left(\partial y^{j}\right)\right)dy^{j}+X^{i}\left(a/\left(\backslash y\backslash\right)\right)e^{2\lambda}dy^{j}\\
&+Y^{i}\left(a/\left(\backslash y\backslash\right)\right)2e^{-2\lambda}\left(\left(\partial\lambda\right)/\left(\partial x^{j}\right)\right)x^{i}dx^{j}-Y^{i}\left(a/\left(\backslash y\backslash\right)\right)e^{-2\lambda}dx^{j}
\end{aligned} \quad (52)$$

And

$$dH=\left(\left(\partial H\right)/\left(\partial x^{j}\right)\right)dx^{j}+\left(\left(\delta H\right)/\left(\delta y^{j}\right)\right)dy^{j}. \quad (53)$$

By means of $i_{XH}\Phi=dH$, the Hamiltonian vector field is found as follows:

$$\begin{aligned}
X_H&=\left(\left(2\backslash y\backslash\right)/a\right)(1/\left(\left[-2e^{-2\lambda}\left(\left(\partial\lambda\right)/\left(\partial x^{j}\right)\right)x^{i}+e^{-2\lambda}+y^{i}2e^{2\lambda}\left(\left(\partial\lambda\right)/\left(\partial y^{j}\right)\right)+e^{2\lambda}\right)\right)\left(\left(\partial H\right)/\left(\partial y^{i}\right)\right)\left(\partial/\left(\partial x^{i}\right)\right)\\
&+\left(\left(2\backslash y\backslash\right)/a\right)\left(1/\left(\left[2e^{-2\lambda}\left(\left(\partial\lambda\right)/\left(\partial x^{j}\right)\right)x^{i}-e^{-2\lambda}-y^{i}2e^{2\lambda}\left(\left(\partial\lambda\right)/\left(\partial y^{j}\right)\right)-e^{2\lambda}\right]\right)\right)\left(\left(\partial H\right)/\left(\partial x^{i}\right)\right)\left(\delta/\left(\delta y^{i}\right)\right).
\end{aligned} \quad (54)$$

Assume that a curve

$$\alpha: I\subset R\rightarrow \tilde{T}^{*}\tilde{M} \quad (55)$$

be an integral curve of the Hamiltonian vector field X, i.e.,

$$X\left(\alpha\left(t\right)\right)=\dot{\alpha}\left(t\right), \qquad t\in I. \quad (56)$$

In the local coordinates, it is obtained that

$$\alpha\left(t\right)=\left(x^{i},y^{i}\right) \quad (57)$$

and

$$\alpha\left(t\right)=\left(\left(dx^{i}\right)/\left(dt\right)\right)\left(\partial/\left(\partial x^{i}\right)\right)+\left(\left(dy^{i}\right)/\left(dt\right)\right)\left(\delta/\left(\delta y^{i}\right)\right). \quad (58)$$

Taking (56), if we equal (54) and (57), it holds

$$\begin{aligned}
&\mathbf{(5)}\left(dx^{i}\right)/\left(dt\right)=\left(\left(2\backslash y\backslash\right)/a\right)\left(1/\left(\left[-2e^{-2\lambda}\left(\left(\partial\lambda\right)/\left(\partial x^{j}\right)\right)x^{i}+e^{-2\lambda}+y^{i}2e^{2\lambda}\left(\left(\partial\lambda\right)/\left(\partial y^{j}\right)\right)+e^{2\lambda}\right]\right)\right)\left(\left(\partial H\right)/\left(\partial y^{i}\right)\right),\\
&\mathbf{(6)}\left(dy^{i}\right)/\left(dt\right)=\left(\left(2\backslash y\backslash\right)/a\right)\left(1/\left(\left[2e^{-2\lambda}\left(\left(\partial\lambda\right)/\left(\partial x^{j}\right)\right)x^{i}-e^{-2\lambda}-y^{i}2e^{2\lambda}\left(\left(\partial\lambda\right)/\left(\partial y^{j}\right)\right)-e^{2\lambda}\right]\right)\right)\left(\left(\partial H\right)/\left(\partial x^{i}\right)\right).
\end{aligned} \quad (59)$$

Hence, the equations introduced in (59) are named conformal Weyl-Hamilton equations on the almost Kähler model $\dot{H}^{*2n}$ of Finsler manifold F^n and then the triple ($\dot{H}^{*2n}$,Φ,X) is said to be a Weyl-Hamiltonian Mechanical System on the almost Kähler model $\dot{H}^{*2n}$ of Finsler manifold F^n. Also, if the above equations (59) λ=0 is selected, the equations [2] are obtained.

$$\begin{aligned}
&\mathbf{(7)}\left(dx^{i}\right)/\left(dt\right)=\left(\left(\backslash y\backslash\right)/a\right)\left(\left(\partial H\right)/\left(\partial y^{i}\right)\right)\\
&\mathbf{(8)}\left(dy^{i}\right)/\left(dt\right)=-\left(\left(\backslash y\backslash\right)/a\right)\left(\left(\partial H\right)/\left(\partial x^{i}\right)\right).
\end{aligned} \quad (60)$$

Mechanical Equations for Conservative Dynamical Systems

Proposition 8: We choose $F=i_{\xi},\ g=\Phi_L and\ f=2\lambda\ at\ i_{\xi}\Phi_L=dE_L$ (31) and by considering the equation $F\left(e^{f}g\right)=F\left(g\right)-df$ (15). Thus, we can write Weyl-Euler-Lagrangian dynamic equation as follows. The second part (31), according to the law of conservation of energy [1], will not change for conservative dynamical systems.

$$i_{\xi}\left(\Phi_L\right)=\Phi_L\left(\xi\right)=dE_L,$$
$$i_{\xi}\left(e^{2\lambda}\Phi_L\right)=\Phi_L\left(\xi\right)-2d\lambda,$$
$$\Phi_L\left(\xi\right)-2d\lambda=dE_L,$$

and $iX_H\Phi_L=d\left(E_L+2\lambda\right).$ from then, added $(L\rightarrow L+2\lambda)$ at the above equation (46). So, we can write

$$\left(\left(\backslash y\backslash\right)/a\right)\left(\partial/\left(\partial t\right)\right)\left(e^{-2\lambda}\left(\left(\partial\left(L+2\lambda\right)\right)/\left(\partial y^i\right)\right)\right)+\left(\left(\delta\left(L+2\lambda\right)\right)/\left(\delta x^i\right)\right)=0,$$

$$\left(a/\left(\backslash y\backslash\right)\right)\left(\partial/\left(\partial t\right)\right)\left(e^{2\lambda}\left(\left(\delta\left(L+2\lambda\right)\right)/\left(\delta x^i\right)\right)\right)-\left(\left(\partial\left(L+2\lambda\right)\right)/\left(\partial y^i\right)\right)=0, \quad (61)$$

The differential equations (61) can be named conformal Weyl-Euler-Lagrangian mechanical system for conservative dynamical systems.

Proposition 9: We choose $F=iX_H$, $g=\Phi$ and f=2λ at (15) and by considering the equation (33)

$$\begin{aligned} &iX_H\Phi=\Phi\left(X_H\right)=dH,\\ &iX_H\left(e^{2\lambda}\Phi\right)=\Phi\left(X_H\right)-2d\lambda,\\ &\Phi\left(X_H\right)-2d\lambda=dH,\\ &iX_H\Phi=dH+2d\lambda=d\left(H+2\lambda\right). \end{aligned} \quad (62)$$

The second part (2), according to the law of conservation of energy for conservative dynamical systems [1], will not change. So, we can write paracomplex conformal Weyl-Hamiltonian dynamic equation as follows:

$$\begin{aligned} &\Phi\left(X_H\right)-2d\lambda=dH,\\ &iX_H\Phi=d\left(H+2\lambda\right). \end{aligned} \quad (63)$$

from then, added (H→H+2λ) at the above equation (46). So, we can write, using (59) and (63), we can obtain

$$\left(dx^i\right)/\left(dt\right)=\left(\left(2\backslash y\backslash\right)/a\right)\left(1/\left(\left[-2e^{-2\lambda}\left(\left(\partial\lambda\right)/\left(\partial x^j\right)\right)x^i+e^{-2\lambda}+y^i2e^{2\lambda}\left(\left(\partial\lambda\right)/\left(\partial y^j\right)\right)+e^{2\lambda}\right]\right)\right)\left(\left(\partial\left(H+2\lambda\right)\right)/\left(\partial y^i\right)\right),$$

$$\left(dy^i\right)/\left(dt\right)=\left(\left(2\backslash y\backslash\right)/a\right)\left(1/\left(\left[2e^{-2\lambda}\left(\left(\partial\lambda\right)/\left(\partial x^j\right)\right)x^i-e^{-2\lambda}-y^i2e^{2\lambda}\left(\left(\partial\lambda\right)/\left(\partial y^j\right)\right)-e^{2\lambda}\right]\right)\right)\left(\left(\partial\left(H+2\lambda\right)\right)/\left(\partial x^i\right)\right). \quad (64)$$

Hence, the equations introduced in (64) can be named conformal Weyl-Hamiltonian Equations for conservative dynamical systems.

Symbolic Solution of the System and Graphic

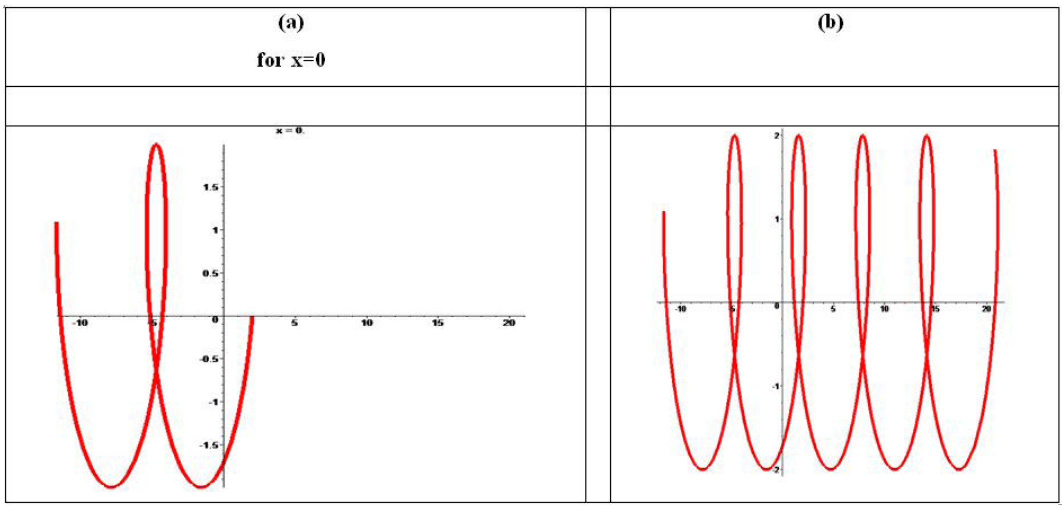

Figure 1: The path followed by Lagrange Graph

It is well-known that an electromagnetic field is a physical field produced by electrically charged objects. How the movement of objects in electrical, magnetically and gravitational fields force is very important. For instance, on a weather map, the surface wind velocity is defined by assigning a vector to each point on a map. Also, each vector represents the speed and direction of the movement of air at that point. The location of each object in space represented by three dimensions in physical space. The equations system (47) and (60) have be solved by using the Maple software and implicit solution is below.

$$I.L(x,y,t) = \exp(t*i)*F_1(y-i*x)+F_2(t)+\exp(-i*t)*F_3(y+x*i),$$

$$for\ \lambda(x,y,t)=0,\ \backslash y \backslash / a = 1, \mathbf{i}^2=-1.$$
$$L(x,y,t) = exp(t*\mathbf{i})+t+exp(-\mathbf{i}*t),$$

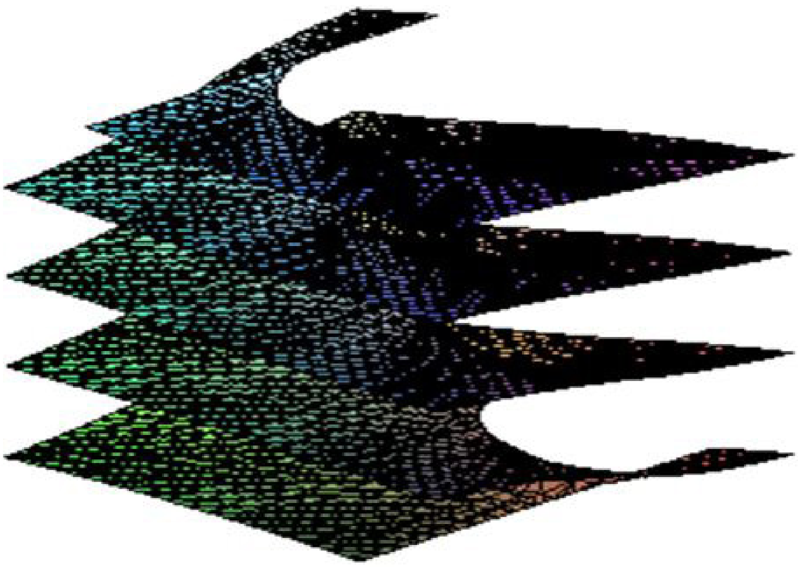

Figure 2: The path followed by the Hamilton Graph

$$II.H(x,y,t) = 1/2*\left(1+\exp(\mathrm{t})^4\right)*\left(\left(y-F(\mathrm{t})\right)*\cos(\mathrm{t})+x*\sin(\mathrm{t})\right)/\exp(\mathrm{t})^2$$
$$for \lambda(x,y,t)=0,\ \mathrm{a}/\|y\|=1, F_4(\mathrm{t})=\mathrm{t}$$

Discussion

By this study, the above-mentioned forms (19) were transferred to the mechanical system for dynamical systems. In addition, in the equations implicit solutions (46) and (59) were found using Maple computation program. So, the Weyl-Euler-Lagrange (46) and Weyl-Hamilton (49) mechanical equations derived on Weyl-Finsler manifolds may be suggested to deal with problems in electrical, magnetically and gravitational fields for the path of movement Figure 1 and Figure 2 of defined space moving objects [32,33]. Therefore, the found equations on Weyl-Finsler geometry has been used in solving problems in different applied areas.

In this study the most important advantage is to obtain geodesic on Weyl-Finsler manifolds. Thus, geodesics is to allow the calculation of linear or nonlinear distance for the orbits of moving objects.

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