Open Access

SCHOLARENA

Journal of Nanoscience and Nanotechnology Applications

RESEARCH ARTICLE

ISSN: 2577-7920

Evolution and Controlling of the Plykin - Newhouse Attractor by the Pyragas Method

Belyakin S[1*], Shuteev S[1] and Kyznetsov S[2]

[1]*Physics Faculty, Department of General Physics, Lomonosov Moscow State University, Moscow, Russia*
[2]*Faculty of Nonlinear Processes, Department of Dynamic Systems, State University of Saratov, Saratov, Russia*

[*]**Corresponding author:** Belyakin S, Physics Faculty, Department of General Physics, Lomonosov Moscow State University, Moscow, Russia, Tel: 8.495.939.51.56, E-mail: bst@newmail.ru; belyakin1962@mail.ru

Citation: Belyakin S, Shuteev S, Kyznetsov S (2018) Evolution and Controlling of the Plykin - Newhouse Attractor by the Pyragas Method. J Nanosci Nanotechnol Appl 2: 203

Article history: Received: 24 January 2018, **Accepted:** 21 May 2018, **Published:** 22 May 2018

Abstract

In the present contribution consideration is being given to an autonomous physical system which is characterized by the presence of the attractor of a hyperbolic type. We study the possibility of controlling and stabilizing the Plykin - Newhouse attractor of this type by the Pyragas method. The choice of the method of control. As such it is possible to use an external signal or the introduction of additional delayed feedback. (Both methods can be realized primarily during the schematic simulation, then in a real experiment). It might also be interesting to think about the realization of a more complicated scheme of control of the type suggested in the work for the stabilization of unstable periodic orbits belonging to the attractor.

Keywords: Nanophysics; Chaotic Systems

Introduction

Hyperbolicity is a fundamental feature of chaotic systems. It is as follows: a tangent space Σ of such systems is a combination of three subspaces; stable E^s, unstable E^u and neutral E^0. Close trajectories which correspond to E^s converge exponentially to each other when $t \to +\infty$, and those which correspond to E^u - when $t \to -\infty$. In the subspace E^0 the vectors contract and expand more slowly than the exponential velocity. When the degree of contraction and expansion in the subspaces E^s and E^u changes from point to point along the trajectory, such systems are called non- uniformly hyperbolic. Dynamic systems with uniform hyperbolicity of all the trajectories are called Anosov systems.

The set Λ is called a hyperbolic attractor of a dynamic system if Λ- is a closed topologic transitive hyperbolic set and there exists such a vicinity $U \supset \Lambda$ that $\Lambda = \bigcap_{t \geq 0} f^n U$. Smale - Williams' solenoid and Plykin's attractor are well- known hyperbolic attractors [1]. Plykin's sphere is obtained by the transformation of the disc domain $T = S^2$ into itself where S^2 - a unitary disc in R^2; then $f: T \to T$, $f(x, y, z) = (\cos\varphi \sin\phi, \sin\varphi \sin\phi, \cos\phi)$ where k>2 determines the compression "by thickness", sets the disc as a subset $T \subset R^3$.

Let there be a smooth family of non - linear controlled systems of ordinary differential equations $\dot{x} = F(x, \mu, u)$, $x \in M \subset R^m$, $\mu \in L \subset R^k$, $u \in U \subset R^n$, $F \in C^\infty$ depending on the vector of controlling parameters u. Suppose that it is necessary to stabilize unstable limiting cycle $x^*(t, \mu^*)$ of the period T, which is the solution of the family when u=0 and $\mu = \mu^*$. Let the system have a regular attractor when the parameters are of the same value u=0, $\mu = \mu^*$. Then the stabilization of the cycle $x^*(t, \mu^*)$ is carried out by means of the feedback with the delay being in the form of $u(t) = \mathrm{K}(x(t) - x(t-T))$, where K - is the matrix of coefficients. Therewith the initial conditional $x(0)$ is chosen in a sufficiently small vicinity of the cycle. Then the solution $x(t)$ of the system $x = F(x(t), \quad , \mathrm{K}(x(t) - x(t-T))$ with the feedback with $\mu = \mu^*$ can converge to the sought - for unstable cycle $x^*(t, \mu^*)$ [2].

Hyperbolic Plykin - Newhouse attractor on a spherical surface

To construct a system with the Plykin - Newhouse attractor let us start with a map of a unit sphere defined as a sequence of four periodically repeating stages of continuous transformations. Duration of each stage is taken to be equal to a unit time interval. The holes will correspond to neighborhoods of four points A, B, C, and D on the spere. Thepicture illustrates the transformations geometrically, and differential equations for all the stages are written down.

$$\dot{x} = \pi y\left(\tfrac{\sqrt{2}}{2}z + \tfrac{1}{2}\right) - \varepsilon xy^2,$$
$$\dot{y} = -\sqrt{2}\pi zx - \tfrac{1}{2}\pi(x+z) + \varepsilon y\left(x^2+z^2\right), \quad (1)$$
$$\dot{z} = \pi y\left(\tfrac{\sqrt{2}}{2}x + \tfrac{1}{2}\right) - \varepsilon zy^2.$$

Figure 1 shows the temporal dynamics of y(t) of hyperbolic Plykin - Newhouse attractor on a spherical surface if $\varepsilon = 0.72$. Figure 2 left shows temporal dynamics of y(t) and the Fourier spectrum on the right presents the temporal dynamics of y(t) and wavelet transform hyperbolic Plykin - Newhouse attractor on a spherical surface if $\varepsilon = 0.72$. Hyperbolic Plykin - Newhouse attractor on a spherical surface in Figure1 and Figure 2 show the bifurcation and chaotic.

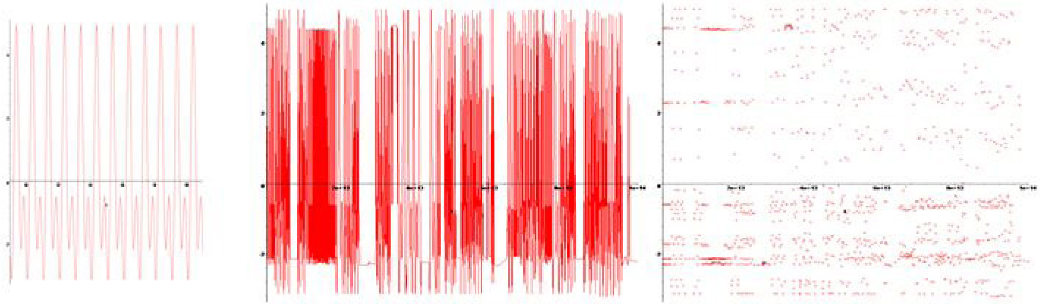

Figure 1: Presents the temporal dynamics of y(t) of hyperbolic Plykin - Newhouse attractor on a spherical surface if $\varepsilon = 0.72$

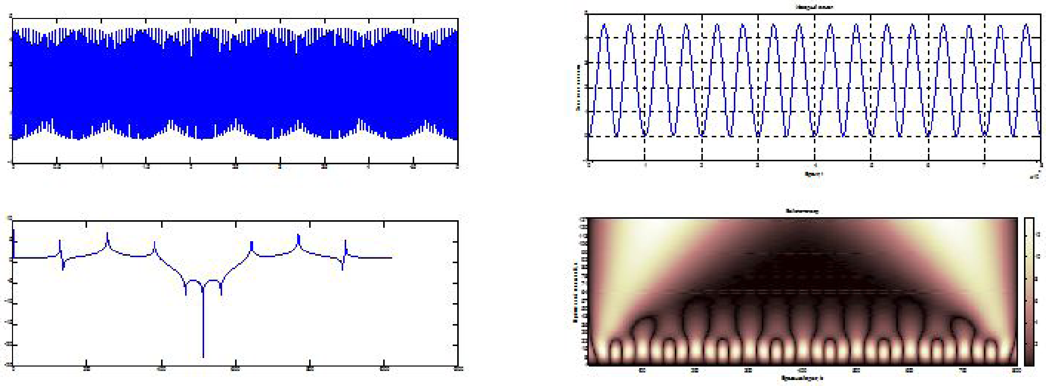

Figure 2: To the left are temporal dynamics of y(t) and the Fourier spectrum on the right presents the temporal dynamics of y(t) and wavelet transform hyperbolic Plykin - Newhouse attractor on a spherical surface if $\varepsilon = 0.72$

We apply the method of Pyragas for hyperbolic Plykin - Newhouse attractor on a spherical surface

For control and synchronization of hyperbolic Plykin - Newhouse attractor on a spherical surface we apply the method of Pyragas.

$$\dot{x} = \pi y\left(\tfrac{\sqrt{2}}{2}z + \tfrac{1}{2}\right) - \varepsilon xy^2,$$
$$\dot{y} = -\sqrt{2}\pi zx - \tfrac{1}{2}\pi(x+z) + \varepsilon y\left(x^2+z^2\right) + 2\pi\mu\left[y(t-\tau) - y(t)\right], \quad (2)$$
$$\dot{z} = \pi y\left(\tfrac{\sqrt{2}}{2}x + \tfrac{1}{2}\right) - \varepsilon zy^2.$$

Figure 3 shows the dynamics and evolution of the attractor Plykin - Newhouse method of Pyragas. In the range μ=1, τ = 0.000025 there is a regular dynamic mode that is observed in the timeline $y(t)$, the Fourier spectrum and wavelet transform.

The transformation of the spherical model in flat system

From representation on the sphere one can pass to the plane by means of the stereographic projection, with the variable change.

$$W = X + iY = \frac{x - z + iy\sqrt{2}}{x + z + \sqrt{2}} \quad (3)$$

As to the attractor of the flow system, it is disposed in the extended phase space (X, Y, and t).

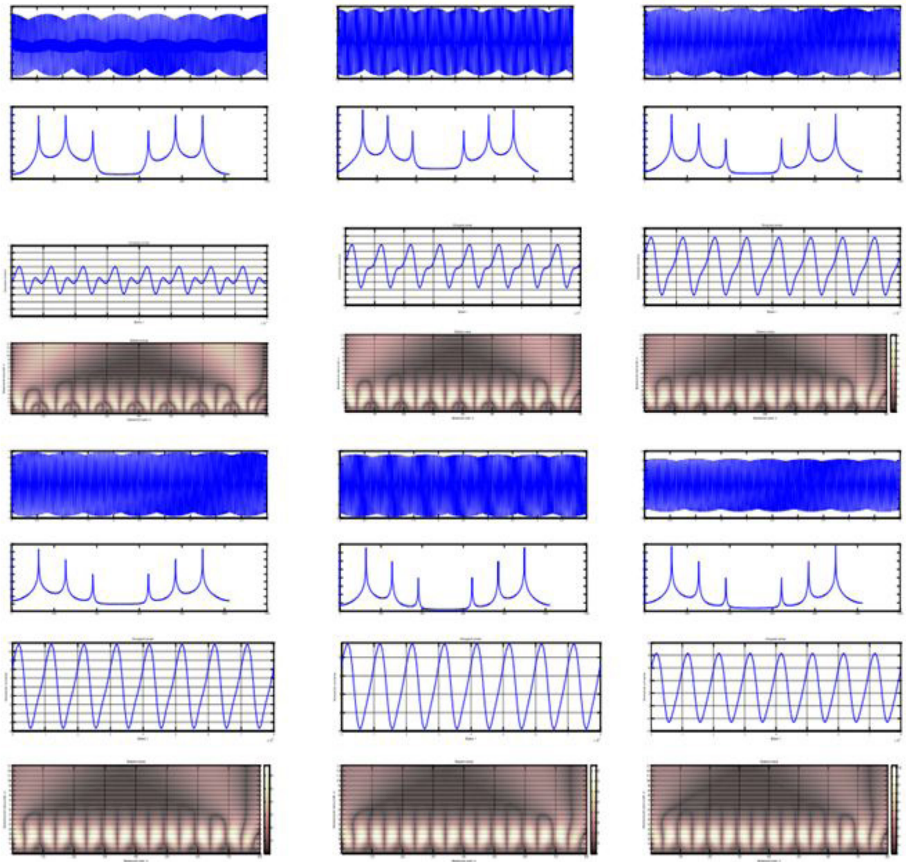

Figure 3: Presents the dynamics and evolution of the behavior of the Plykin - Newhouse attractor under the action of a method of Pyragas shown when $\varepsilon = 0.72$, $\tau = 2.5 \cdot 10^{-5}$, $\mu = 0.25,\ 0.5,\ 0.75,\ 1.0,\ 1.25,\ 1.5$

The use of the Pyragas method for the formation of regular dynamics in autonomous hyperbolic attractors

Let us take into consideration the system of the type [3].

$$\begin{aligned}
&\dot{X} = -2\varepsilon Y^2\Omega_1\left(\cos(\omega_2\cos\omega_1 t) - X\sin(\omega_2\cos\omega_1 t)\right) + kY\Omega_2\left(\cos(\omega_2\cos\omega_1 t) - X\sin(\omega_2\sin\omega_1 t)\right)\sin\omega_1 t,\\
&\dot{Y} = 2\varepsilon Y\Omega_1\left(X\cos(\omega_2\cos\omega_1 t) - \tfrac{1}{2}(1-X^2+Y^2)\sin(\omega_2\cos\omega_1 t)\right) -\\
&\quad k\Omega_2\left(\cos(\omega_2\sin\omega_1 t) + \tfrac{1}{2}(1-X^2+Y^2)\sin(\omega_2\sin\omega_1 t)\right)\sin\omega_1 t + D_{Y,\tau},\ \ D_{Y,\tau} = K[Y(t-\tau) - Y(t)],\\
&\Omega_1 = \frac{2X\cos(\omega_2\cos\omega_1 t) + (1-X^2-Y^2)\sin(\omega_2\cos\omega_1 t)}{(1+X^2+Y^2)^2},\\
&\Omega_2 = \frac{-2X\sin(\omega_2\sin\omega_1 t) + (1-X^2-Y^2)\cos(\omega_2\sin\omega_1 t)}{1+X^2+Y^2} + \frac{\sqrt{2}}{2}.
\end{aligned} \tag{4}$$

Here *X*, *Y* - dynamic variables, ε & k - coefficient of connection, - $\omega_{1,2} = \left(\pi/2, \pi/4\right)$ inherent frequency oscillations. The phase portraits and Fourier spectrums presented demonstrate the behavior of the Plykin - Newhouse system. K=0 corresponds to the chaos Figure 4; Figure 5 corresponds to the stable state when K=1.8 and τ=1.8.

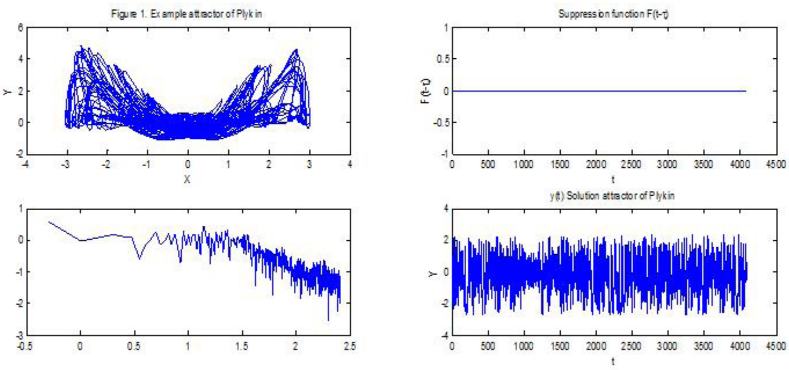

Figure 4: On the left a phase portrait of the Plykin - Newhouse system and corresponding spectral density, And on the right the dynamic variable is represented on the scale of time are shown when $\varepsilon = 0.72$, $k = 1.9$

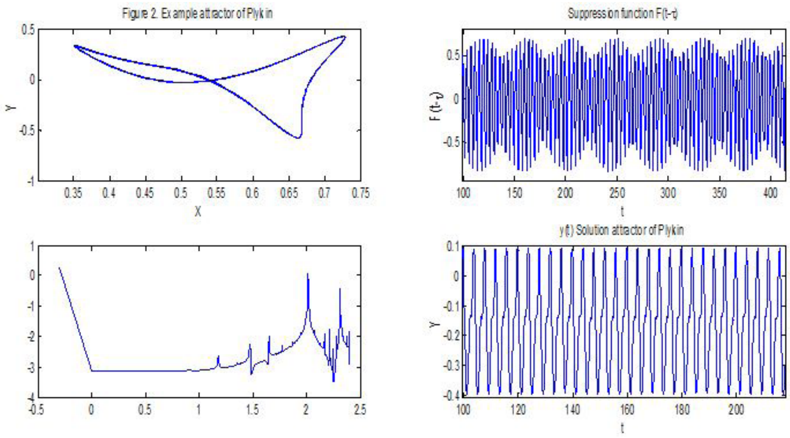

Figure 5: The Plykin - Newhouse system corresponding to the spectral density is shown when, $\tau = 1.8,\ K = 1.8,\ \varepsilon = 0.72,\ k = 1.9$

Make the transformation with the flat system back to the spherical model

For transformation of flat system back to the spherical model we use the system of equations (5).

$$x = \frac{1 - X^2 - Y^2 + 2X}{\sqrt{2}\left(1 + X^2 + Y^2\right)},\ \ y = \frac{2Y}{1 + X^2 + Y^2},\ \ z = \frac{1 - X^2 - Y^2 - 2X}{\sqrt{2}\left(1 + X^2 + Y^2\right)} \quad (5)$$

Figure 6 shows on the left the temporal scale amplitude *y(t)*, to the right of the phase portrait of *y(x)*, in the absence of external influence on the attractor. It is easy to see that, at these parameter values the attractor has strong chaotic properties.

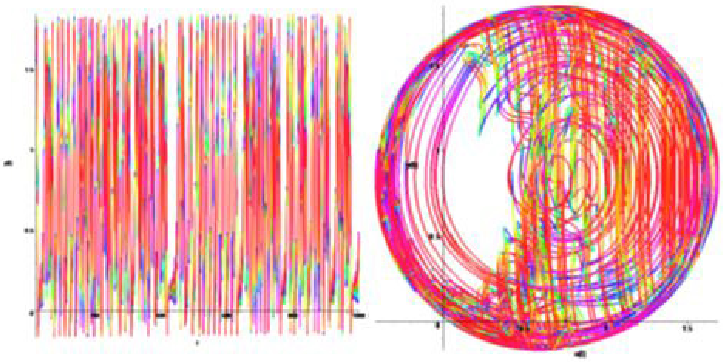

Figure 6: Timeline *y(t)* and phase portraits *y(x)* of the attractor Plykin - Newhouse

Result

When using the method of Pyragas $K = 0.1 \to 1.0,\ \tau = 1.9,\ \varepsilon = 0.72,\ k = 1.9$ is observed evolutionary dynamics of phase portraits of the system shown in Figure 7.

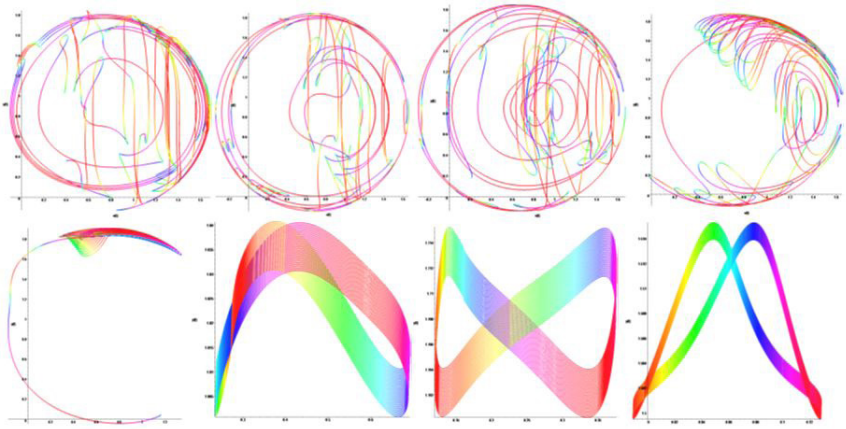

Figure 7: Phase portraits of y(x) of the Plykin - Newhouse attractor when $K = 0.1 \to 1.0,\ \tau = 1.9,\ \varepsilon = 0.72,\ k = 1.9$

A positive impact on the attractor the attractor translates the pole on the positive sector of the equator. Attractor points *A, B, C* and *D* closer together forming a stable periodic attractor.

The stereographic projection

The stereographic projection is a particular mapping that allows us to project a sphere onto a plane. The original projection is defined on the entire sphere, but we use in this study a version restricted to the hemisphere. This mapping is bijective and preserves the angles: a Cartesian grid on the plane corresponds to a Cartesian grid on the surface of the sphere. However, this particular mapping is neither isometric nor area-preserving: the distances and the areas are not preserved. In Cartesian coordinates, if one denotes the coordinates on the sphere by (*x, y, and z*) and the coordinates on the plane by (*X, Y*), the projection and its inverse are given by the formula (6) [4]:

$$\begin{gathered} W = X + iY = \frac{x(1+z) + iy(1+x)}{(1+x)(1+z)}, \\ (x,y,z) = \left(\frac{2X}{1+X^2+Y^2}, \frac{2Y}{1+X^2+Y^2}, \frac{1-X^2-Y^2}{1+X^2+Y^2} \right). \end{gathered} \tag{6}$$

The radius of the sphere is equal to one in these formulas and in the sequel. Stereographic projection: *N* and *S* denote respectively the north and south poles, the points (*A, B, D*) on the sphere is projected on *C* on the plane.

As to the attractor of the flow system, it is disposed in the extended phase space (*X, Y, and t*) (Figure 8).

$$\begin{gathered} \dot{X} = \frac{Y}{1+X^2+Y^2}\left(\frac{\pi}{4}\left[1+\sqrt{2}-(\sqrt{2}-1)X^2-(\sqrt{2}-1)Y^2 \right] - \varepsilon Y \right), \\ \dot{Y} = -\sqrt{2}\,2\pi \frac{X\left(1-X^2-3Y^2\right)}{1+X^2+Y^2} - \frac{\pi}{2}\left(1-2X-X^2+\frac{3}{2}Y^2 \right) + \varepsilon 2Y\left(1-\frac{4Y^2\left(X^2+Y^2\right)}{\left(1+X^2+Y^2\right)^2} \right). \end{gathered} \tag{7}$$

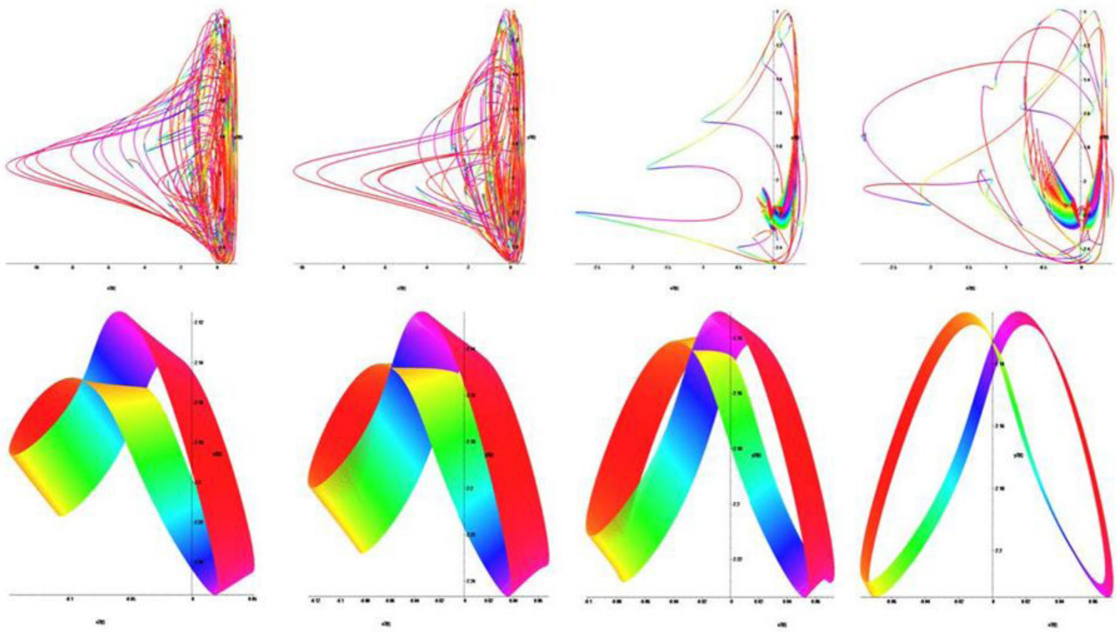

Figure 8: Phase portraits of Y(X) of the Plykin - Newhouse attractor when $K = 0.0 \to 0.4,\ \tau = 1.9,\ \varepsilon = 0.72,\ k = 1.9$

For transformation of flat system back to the spherical model we use the system of equations (6).

Figure 9 shows on the left the temporal scale amplitude *y(t)*, to the right of the phase portrait of *y(x)*, in the absence of external influence on the attractor. It is easy to see that, at these parameter values the attractor has strong chaotic properties.

Result

When using the method of Pyragas $K = 0.0 \to 0.4,\ \tau = 1.9,\ \varepsilon = 0.72,\ k = 1.9$ is observed evolutionary dynamics of phase portraits of the system shown in Figure 9.

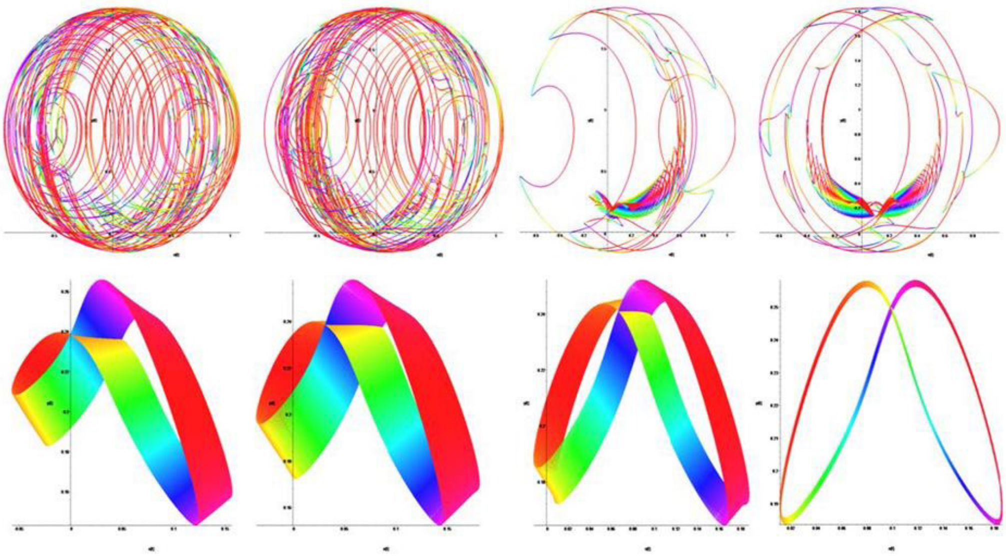

Figure 9: Phase portraits of *y(x)* of the Plykin - Newhouse attractor when $K = 0.0 \rightarrow 0.4,\ \tau = 1.9,\ \varepsilon = 0.72,\ k = 1.9$.

Conclusion

And the hyperbolic attractor degenerates into the limiting cycle, and the continuous spectrum corresponding to chaotic oscillations changes into an equidistant one with the frequencies corresponding to the basic frequency and its harmonics. Thus, the application of the method of Pyragas at a constant time delay, gives the opportunity to observe the evolutionary dynamics of systems of hyperbolic Plykin - Newhouse attractor.

References

1. Belyakin ST, Dzhanoev AR, Kuznetsov SP (2014) Stabilization of Hyperbolic Chaos by the Pyragas Method. J Math System Sci 4: 755-62.

2. Pyragas K (2001) Control of chaos via an unstable delayed feedback controller. Phy Rev Lett 86: 2265-8.

3. Kuznetsov SP (2009) An example of a non-autonomous continuous-time system with attractor of Plykin type in the Poincare map. Nonlinear Dyn 5: 403-24.

4. Xiong YL, Fischer P, Bruneau CH (2012) Numerical simulations of two-dimensional turbulent thermal convection on the surface of asoap bubble. Seventh International Conference on Computational Fluid Dynamics (ICCFD7) 7: 1-8.