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Estimating Exposures for a Mortality Study of a National Pension Scheme

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Abstract

Information on the number of lives exposed to a particular risk provide a measure of responsiveness of the studied policies to the event and also help in planning, particularly in managing a pension scheme. In this paper, the actuarial exposure method of computing exposed to risk of death is explored and applied to obtain the amount of exposures contributed to each unit interval by each member of a national pension scheme observed over a period of 10 years. The results indicate that the ages of scheme members ranged between 18 and 110 years and those members between 30 and 60 years old were mostly vulnerable to risk of death. Generally, the crude mortality rates of scheme members exhibited random upward fluctuations trend with increasing age. However, the mortality rates of the young scheme members (aged 18-49 years old) were more dispersed compared with the adults (aged 50-110 years old), whose mortality rates spread evenly among the ages. The crude mortality rates coupled with fluctuating trend are unsuitable for further actuarial purposes such as annuity pricing. However, the study serves as a baseline for a companion paper, which seeks to extend the methodology in this study by employing a non-parametric graduation technique to smooth the crude mortality rates for further insurance practices in Ghana.

Keywords: Exposures (Exposed to risk); Crude Mortality Rates; Actuarial Exposure Method; Age/Duration Vector; Pension Scheme

Introduction

Pension mortality studies are done to provide current information across pension funds for use in setting up a plan for actuarial valuation. However, due to difficulty in acquiring the needed data, pension mortality studies have historically not been performed often relative to life insurance mortality studies. The observed data for constructing mortality tables include the number of deaths and exposures at specific age intervals. However, only a sample from the population exposed to the decrement death experienced the event. The exposure is defined as the number of units of human life that are subject to the risk of death, incidence of disability, lapse or any other form of decrement within a pre-determined observation period [1-2]. These events affect insurance policies and thus can be the subject of an experience study, an exercise that evaluates the occurrences of specific events including deaths and incidence of disability lapses within a defined time frame and pertains to a given population [3]. Another concept which is of a great interest when conducting experience studies is the use of insuring ages. Most individuals do not buy insurance policies on their very birthday and hence tend to have fractional ages as their dates of entry into a study. However, for convenience sake, many life insurance companies assign integral ages to a policyholder's true age. The idea of replacing a policyholder's actual birthday with the date at which he or she purchases the policy is known as insuring ages. It is a common practice to find life insurers assign policyholders their age nearest birthday. Only few life insurers prefer to assign policyholders their age last birthday. When an anniversary-to-anniversary study is combined with insuring ages, all lives enter the study at integer ages. Mahler [4] observed that when the age nearest birthday is applied to a study, the insuring age becomes an unbiased estimator of the actual age, which can lead to an absolute error of up to half a year; though this affects premiums by 2 - 2.5%, an insurer is presumably willingly to accept this in return for the convenience of applying the insuring age. On the contrary, when the age last birthday is applied, the insuring age rather becomes a biased estimator of the actual age and there can be an absolute error of up to a year, lowering premiums to about 5%.

Having information about the number of lives exposed to a particular risk provide a measure of responsiveness of the studied policies to the event and also help in planning a pension fund. Thus, before any actuarial work in relation to life insurance is considered, a proper analysis of mortality including the number exposed to risk must be conducted. The commonly techniques employed in calculating exposures include the exact exposure and actuarial exposure methods. The difference between the two methods centers on the date at which exposure to the risk of death as a decrement should cease. However, the latter method is much preferred to the former due to its simplicity and ease in its application to large dataset. Hoem [5] argued that the theory underlying the conventional actuarial method of estimating mortality rates is biased and inconsistent, leading to a flaw in the use of actuarial exposed to risk computation. Nearly a decade later, Dorrington & Slawski [6] defended the use of the actuarial method by refuting the inconsistency property as proposed by Hoem [5]. In this paper, the actuarial method of computing exposed to risk is used to calculate the number of units of lives exposed-to-risk of death and compared with number of deaths within using data from a national pension scheme in Ghana.

Methods

In order to estimate exposures and subsequently the crude mortality rates, the exact age approach is used to obtain actual and exact ages of lives at specific times during an observation period. The information required on each member of the pension scheme to undergo this estimation were the date of joining the scheme, date of birth, and date of death or withdrawal, which includes any random decrement other than death. Following London [7], the estimation interval $(x, x+1]$ is chosen to determine each life's contribution to each unit interval, where $x = 18, 19, \ldots, 110$ are the ages (in years) of the pension scheme members. The interval $(x, x+1]$ is much preferred because it resonates with the definition of the mortality rate (q_x) which is the probability that a life dies within one year given that life is alive at age x [7]. The exact age at which a death occurs is obtained by deducting the decimal year of birth from the decimal year of the death. Then for each life i observed in the pension scheme, we define the following notations:

- y_i, denotes the exact age of life i into the study;
- z_i, denotes the exact scheduled age of life i out of the study;
- θ_i, denotes the exact age at death ($\theta_i = 0$ if life i did not die during the observation period);
- φ_i, denotes the exact age at withdrawal ($\varphi i = 0$, if life i did not withdraw during the observation period).

Furthermore, each life i is assigned the age vector $v_i' = [y_i, z_i, \theta_i, \varphi_i]$, which defines the exact ages at which the various deaths occur. The dataset provided by the age vector is needed to calculate each life's contribution to the age interval $(x, x+1]$. To determine life i's contribution, if any, to the age interval $(x, x+1]$, we note that when [8]:

1. $y_i \geq x+1$, life i does not contribute to the interval $(x, x+1]$.
2. $z_i \leq x$, life i does not contribute to the interval $(x, x+1]$ since life is exit from the study is before age x.
3. $y_i \leq x$ and $z_i \geq x+1$, life i is scheduled to be under observation for the entire interval $(x, x+1]$.
4. $x < y_i < x+1$ or $x < z_i < x+1$ or both, life i can be said to enter the estimation interval $(x, x+1]$ at age $x + r_i$ and exit at age $x + s_i$ where r_i and S_i represent the interval of age within $(x, x+1]$ over which life i potentially comes under observation and potentially exits from an observation, respectively.
5. $y_i < x+1$ and $z_i > x$, then life i is also scheduled to contribute to the age interval $(x, x+1]$.

After the above conditions have been duly observed, each age vector is converted into a duration vector $u_i' = [r_i, s_i, l_i, k_i]$, for a given life i during the age interval $(x, x+1]$. The duration vector u_i' shows the fractional duration within the unit interval at which point the observation commences, is scheduled to end, or actually ends due to death or withdrawal before the scheduled end date. The components of vector u_i' are defined given by equations (1)-(4):

$$r_i = \begin{cases} 0 & \text{if } y_i \leq x \\ y_i - x & \text{if } x < y_i < x+1 \end{cases} \quad (1)$$

$$s_i = \begin{cases} z_i - x & \text{if } x < z_i < x+1 \\ 1 & \text{if } z_i \geq x+1 \end{cases} \quad (2)$$

$$l_i = \begin{cases} 0 & \text{if } \theta_i = 0 \\ \theta_i - x & \text{if } x < \theta_i \leq x+1 \\ 0 & \text{if } \theta_i > x+1 \end{cases} \quad (3)$$

$$k_i = \begin{cases} 0 & \text{if } \varphi_i = 0 \\ \varphi_i - x & \text{if } x < \varphi_i < x+1 \\ 0 & \text{if } \varphi_i > x+1 \end{cases} \quad (4)$$

The exact exposure over $(x, x+1]$ contributed by life i is given by the minimum of the components s_i, l_i and k_i of u_i' that exceeds 0 minus r_i. Thus, the actuarial exposure over the age interval $(x, x+1]$ contributed by life i at age x, is defined as:

$$e_{i,x} = min(s_i, k_i, 1) - r_i, \quad (5)$$

where i=1,2,3,...,n, and the components s_i, k_i and 1 are used if the life i does not die or withdraw; life i withdraws; and life i dies, respectively. Then, the total actuarial exposure contributions over the interval (x, x+1] for all the lives at age x is given by equation (6):

$$E_x = \sum_{i=1}^{n} e_{i,x} \qquad (6)$$

For instance, given the age vector [37.5,43.1,41.4,0] for a life i who was studied from May 1, 2002 to December 31, 2006, the actuarial exposure at each age x is given by Table 1, where each row indicates the exposure contribution to the interval (x,x+1]. It should be noted that $u_{i,x}$ is not defined for $x > 42$ and $x < 37$, since life i does not contribute to these intervals.

Age (x)	s_i	l_i	k_i	r_i	$e_{i,x}$
37	1	0	0	0.5	0.5
38	1	0	0	0	1
39	1	0	0	0	1
40	1	0	0	0	1
41	1	0.44	0	0	1

Table 1: Actuarial exposure contribution at age within (x, x+1]

The age of the pension scheme member ranges from 18 to 110 years. Often, the exposure E_x is considerably large, making the number of deaths and proportion of deaths follow the binomial distribution. The annual crude mortality rate is estimated by equation (7):

$$q_x = \frac{d_x}{E_x} \qquad (7)$$

where:

- $d_x = \sum_{i=1}^{n} d_{i,x}$ and $d_{i,x}$ is an indicator variable defined by:

$$d_i = \begin{cases} 1 & \text{if the life } i \text{ dies at age } x \\ 0 & \text{otherwise} \end{cases} \qquad (8)$$

- $e_{i,x}$ is the exposure contributed by the life i to the age interval [x, x+1), as defined in equation (5);
- d_x is the total number of deaths within the interval [x, x+1);
- E_x is the total exposure within the interval [x, x+1), as defined in equation (6).

Results and Discussions

The data for the study were obtained from Ghana's largest pension fund, which serves as the main driver of the development of the capital market in the country. The national pension fund has about 1.31 million active members with over 184,761 pensioners who regularly receive monthly pensions while annual growth of retirements stands about 12,000 [9]. Membership contributions account for about 33% of its total source of funds while sustainability of the pension scheme relies mostly on the numerous investments the Fund has undertaken over the years [10]. The data collected included birth dates, policy issue dates, withdrawal dates and dates of death of scheme members, spanning over the period 2005-2015. The number of deaths for each age x was recorded while the actuarial exposure method was employed to compute the number of exposures for each age, following equations (1)-(6). Once the exposures have been computed, the annual crude mortality rates (q_x) were easily computed as the ratio of the number of deaths (d_x) to the actuarial exposure at each year of age x (E_x) per equations (6)-(8).

Descriptive Statistics

The summary statistics of ages of the members of the scheme (x), coupled with the number of deaths, exposures and crude mortality rates per age (x) are presented in Table 2, while Figure 1 displays their distributions graphically. The ages of the members of the pension scheme observed were distributed between 18 and 110 years inclusive with the mean and median having the same value of 64 years. The computed number of exposures appear more variable, ranging from 130 to 411,953 with a mean of 157,116.8 per age x while the number of deaths recorded for each age x averages 4,556 and lies within the range 56-23,017. Most of these deaths occurred among the scheme members aged between 55 and 75 years. The crude mortality rates varied from 0.0005 to 0.48429 with a mean value of 0.12904 per age x, exceeding the median (0.06573) by half.

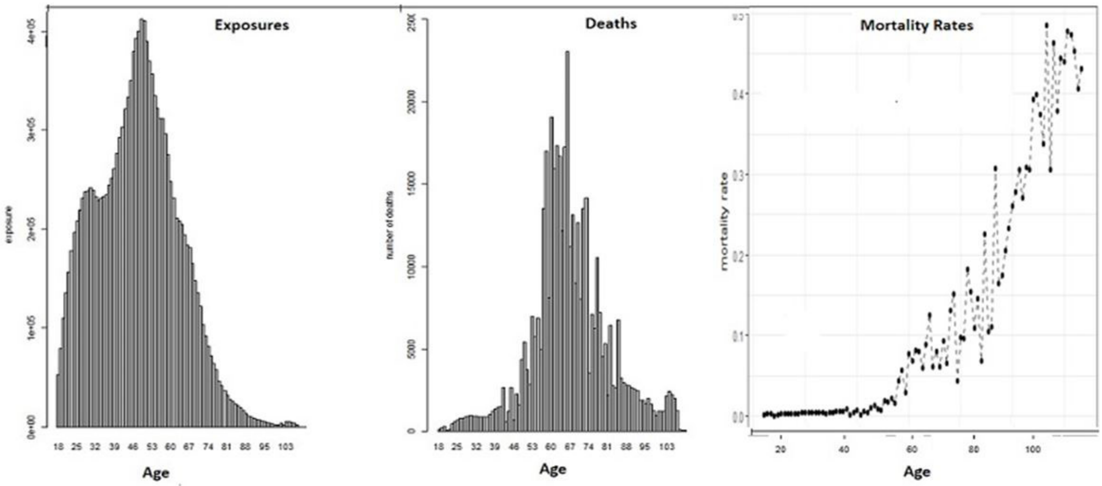

Figure 1: The distributions of exposures, number of deaths and crude mortality rates per age of pension scheme members

Estimation of Exposures and Crude Mortality Rates

The estimated number of members exposed to risk of death or exposures (E_x) and the crude mortality rates (q_x) at each age (x) were further analysed for future smoothing of q_x required in pricing of insurance products. The detailed results from the computations are summarized in Table 3 and graphically displayed in Figures 1 and 2. Table 3 gives the computed exposures, number of deaths and crude mortality rate at each age (x) while Figure 1 provides the plots of the exposures (E_x), number of deaths (d_x) and the crude mortality rate (q_x) for each age (x).

It is observed that large number of the exposure are captured between ages 30 and 60 years but gradually decreases thereafter (Table 3 and Figure 1). There was a significant increase in the exposed to risk of death for members aged from 18 to 30 years, followed by a slight decrease for further few years before it begun to increase steadily until reaching its peak at age 49 years. It then decreased exponentially until it reached its lowest values beyond age 100 years. The increase in exposures at early ages (18-50 years) could be attributed to newly employed job seekers who had just joined and started contributing to the scheme, while the decrease within the same age range could be blamed on numerous deaths due to inherent youthful exuberance activities, which are hardly controlled in many communities in Ghana. The gradual decrease from the peak at older ages (50+ years) could also be attributed to factors such as poor health condition, neglect and stress leading to deaths thereby reducing the number of people and contributing to the reduction in exposures. The crude mortality rates of age (x) of the pension scheme, which were also computed, produced very low and seemingly stable number of deaths relative to the exposures at ages between 18 and 50 years. Thereafter, the crude mortality rates begun to rise and fall in an upward trend throughout. The drops in mortalities could also be explained by the lack of data and misreporting of deaths at such ages while the observed fluctuating patterns in the mortality of members could be resolved by smoothing to make the mortality rates more suitable for pricing life insurance products. This task has been accomplished in the companion paper on graduation of the crude mortality rates.

Variable	Minimum	1st Quartile	Median	Mean	3rd Quartile	Maximum
Age (x)	18	41	64	64	87	110
Ex	130	16,924	157,411	157,611.8	244,398	411,953
dx	56	978	2291	4556	6394	23017
qx	0.00050	0.00423	0.06573	0.12904	0.22578	0.48429

Table 2: Summary statistics of age x, exposures (E_x), number of deaths (d_x) and crude mortality rates (q_x) per age (x)

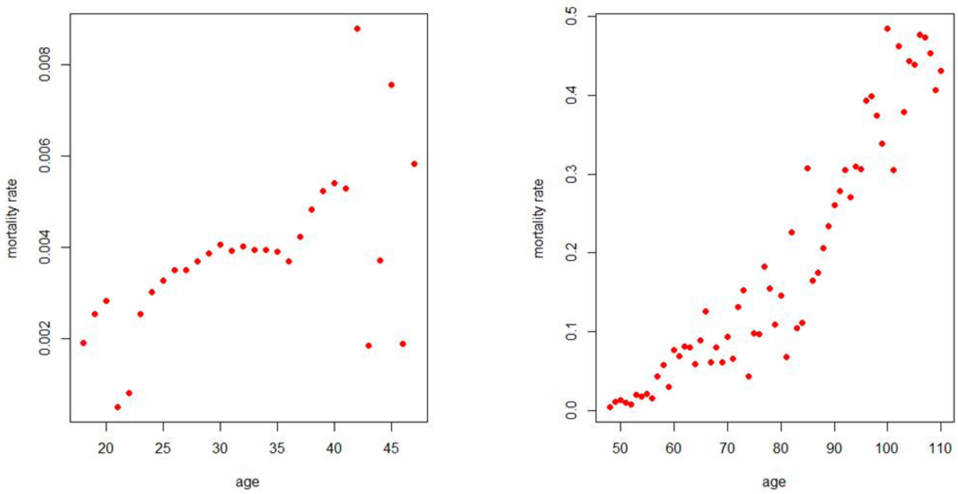

Figure 2: Crude mortality rates of scheme members at ages 18-49 years (left) and at ages 50-110 years (right)

Age (x)	E_x	d_x	Crude q_x	Age (x)	E_x	dx	Crude q_x
18	53202	101	0.00190	65	194191	17209	0.08862
19	79457	203	0.00255	66	183830	23017	0.12521
20	109951	311	0.00283	67	181923	11187	0.06149
21	136191	68	0.00050	68	165244	13165	0.07967
22	157411	127	0.00081	69	147811	8984	0.06078
23	178241	452	0.00254	70	135992	12676	0.09321
24	197029	597	0.00303	71	122158	8030	0.06573
25	208165	683	0.00328	72	103355	13523	0.13084
26	219348	770	0.00351	73	93055	14157	0.15214
27	231056	810	0.00351	74	82179	3547	0.04316
28	237765	877	0.00369	75	72130	7082	0.09818
29	238302	920	0.00386	76	64476	6227	0.09658
30	241460	978	0.00405	77	57544	10512	0.18268
31	239152	937	0.00392	78	46712	7201	0.15416
32	232922	937	0.00402	79	41789	4578	0.10955
33	228976	903	0.00394	80	36492	5316	0.14568
34	231089	910	0.00394	81	32313	2200	0.06808
35	232832	909	0.00390	82	28320	6394	0.22578
36	234803	869	0.00370	83	26169	2727	0.10421
37	244398	1034	0.00423	84	23902	2653	0.11099
38	251518	1215	0.00483	85	22032	6771	0.30733
39	261093	1365	0.00523	86	19689	3251	0.16512
40	276836	1498	0.00541	87	16924	2950	0.17431
41	292028	1544	0.00529	88	13618	2802	0.20576
42	302599	2658	0.00878	89	11820	2757	0.23325
43	321103	594	0.00185	90	10241	2666	0.26033
44	333944	1241	0.00372	91	9177	2550	0.27787
45	350148	2643	0.00755	92	8021	2450	0.30544
46	378879	712	0.00188	93	7037	1905	0.27072
47	393434	2291	0.00582	94	6216	1922	0.30918
48	400598	1613	0.00403	95	5632	1725	0.30627
49	411953	4360	0.01058	96	5088	2002	0.39349
50	410088	5441	0.01327	97	4195	1673	0.39879
51	389476	3709	0.00952	98	3408	1274	0.37378
52	369748	2862	0.00774	99	2908	983	0.33804
53	356793	6966	0.01952	100	2527	1224	0.48429
54	335427	5770	0.01720	101	4031	1231	0.30538
55	321909	6853	0.02129	102	2678	1239	0.46266
56	312045	4979	0.01596	103	5668	2145	0.37844
57	311445	13511	0.04338	104	5405	2397	0.44348
58	295933	17001	0.05745	105	5102	2240	0.43904
59	275918	8136	0.02949	106	4221	2014	0.47714
60	248337	19062	0.07676	107	2710	1283	0.47350
61	230794	15959	0.06915	108	183	83	0.45281
62	211945	17303	0.08164	109	160	65	0.40650
63	208319	16750	0.08041	110	130	56	0.43044
64	205396	12194	0.05937	-	-	-	-

Table 3: Estimated exposures together with number of deaths and crude rates at each age *x*

The crude mortality rates for the young (18-49) and adult (50-110) year-olds are graphed separately (Figure 2) for clear visible behaviour of mortality rates of young and old scheme members. It is observed from the graphs that the crude mortality rates are more dispersed at ages 18-23 years and 41-47 years but remain relatively stable and consistent in between these age groups. However, the mortality rates observed in the adult ages are evenly and consistently dispersed with visible upward trend (Table 3).

Conclusion

Modelling and managing mortality risk are of major concerns of pension fund schemes in view of their growing realization in recent times [11]. The study applied the actuarial exposure method to a case study of a national pension scheme in Ghana. The data collected covered a period of over 10 years on members of the national pension scheme and whose ages varied from 18 to 110 years. It was found that members whose ages ranged from 30 to 60 years were mostly vulnerable risk of death, accounting for more than half of the total exposures. The crude mortality rates, computed as ratio of number of deaths to the exposures at all ages, generally exhibited an increasing trend with random fluctuations. However, the mortality rates at lower ages were relatively very low and widely scattered.

In conclusion, the number of exposures and mortality rates at each age level of a national pension fund members over a period have been tabulated (Table 3). To achieve mortality rates that progressed smoothly as age increased for actuarial purposes, non-parametric graduation techniques such as the Whittaker-Henderson methods could be explored to smooth the crude rates [12-15]. Future work being considered is a companion paper, which seeks to extend this study by graduating the crude mortality rates for actuarial use such as annuity pricing.

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